A systematic review of the use of ICTs in developing pupils' understanding of algebraic ideas

Review conducted by the Mathematics Education Review Group

Technical report written by Maria Goulding and Chris Kyriacou

EPPI-Centre
Social Science Research Unit
Institute of Education
University of London

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TECHNICAL REPORT

The results of this systematic review are available in four formats:

SUMMARY	Explains the purpose of the review and the main messages from the research evidence
REPORT	**Describes the background and the findings of the review(s) but without full technical details of the methods used**
TECHNICAL REPORT	Includes the background, main findings, and full technical details of the review
DATABASES	Access to codings describing each research study included in the review

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CONTENTS

List of abbreviations

ABI	Autograph-based instruction
AEI	Australian Education Index
BEI	British Education Index
BSRLM	British Society for Research into Learning Mathematics
CAS	Computer algebra systems
DfES	Department for Education and Skills
DG	Dynagraph
ERIC	Educational Resources Information Center
GC	Graphics calculator
ICTs	Information and communication technologies
ILP	Individual / integrated learning programme
ILS	Independent/Individual Learning Systems
IWB	Interactive white board
KS2	Key stage 2 (ages 7-11 years)
KS3	Key stage 3 (ages 11-14 years)
KS4	Key stage 4 (ages 14-16 years)
MAT	Mathematics achievement test
NC	National Curriculum for England
NCETM	National Centre for Excellence in Teaching Mathematics
NS for KS3	National strategy for key stage 3
OHP	Overhead projector
REEL	Research Evidence in Education Library
SBI	Spreadsheet-based instruction
TBI	Traditional-based instruction
TDA	Training and Development Agency for Schools
WoE	Weight of evidence
WWC	What Works Clearing House

Abstract

The review question

The review set out to answer the review question:

> *How have different information and communication technologies (ICTs) contributed to the development of understanding of algebra for pupils up to the age of 16?*

After keywording the question was narrowed down to the following:

How have different information and communication technologies (ICTs) contributed to the development of understanding of functions for pupils up to the age of 16 (with particular reference to the relationships between different representations and the interpretation of graphical representations)?

Who wants to know and why?

This review is set in the context of the National Strategies for primary and secondary education in England and Wales, which are both part of the drive to raise standards in schools. The use of a range of ICTs is encouraged by these strategies in the expectation that effective use of ICTs, which requires substantial funding, will raise standards. The review has been commissioned by the Training and Development Agency for Schools (TDA), which has already commissioned reviews into the use of ICTs in English and Science. This review will focus upon Mathematics and, in particular, a crucial part of the algebra curriculum, that of functions. The TDA were particularly interested, not just in whether ICTs could contribute to the development of understanding of functions, but also under what conditions that understanding developed. Others involved in policy, practice and research in mathematics education in England and Wales also need to know what the best quality international research can offer to inform teaching with ICT in this aspect of the mathematics curriculum.

Methods of the review

Identifying relevant studies involved carrying out an electronic search using keywords with bibliographic databases, hand-searching conference proceedings, citations and publications recommended by contacts. This resulted in 33 studies being identified for the systematic map and 14 for the in-depth review.

Results

The studies in the in-depth review give us statistical evidence of gains in understanding as a result of interventions incorporating ICTs, evidence of the nature of these understandings, evidence of some common difficulties experienced when using graphical calculators, and detail of ways of working in the interventions.

Gains in understanding

Three studies give evidence of general gains in interventions, each using one type of ICT. One study indicates that pupils working in the computer medium performed better than those in the paper and pencil medium, although both made gains in graphical interpretation. One study evidences differences in gains according to the type of software, and, importantly, that an intervention not incorporating technology was more effective than one of the interventions incorporating ICT. One study gives evidence of gains according to the type of software, and, importantly, that an intervention not incorporating technology was more effective than one of the interventions incorporating use of a spreadsheet; in this case, the pupils had been taught how to use the spreadsheet but not in a mathematical context. This points to the importance of the design of the software and the way it is introduced.

Nature of understanding

There is evidence of some students successfully using visualisation with graphing software to fit graphs to datasets, to solve equations and to transform functions. In terms of interpreting graphs of rates of change, there is evidence that pupils working in a computer environment reached higher levels of thinking and were able to explain their thinking better than pupils working in a paper and pencil medium. There is also some evidence of lower attaining students preferring to work arithmetically with tables of values and only later moving to integrate the tables of values with computer generated graphs. There is also some evidence of pupils having difficulty with moving between symbolic, tabular and graphical forms when solving equations. Some of these differences may be accounted for by differences in the tasks and whether the tasks were context free or contextualised.

Difficulties of working with graphics calculators

There is evidence that students do not always know how to use the technology, interpret ambiguities in the output and exercise critical judgment when using some of the facilities of advanced calculators. These studies are of relevance to the review question, because they show that the learner has to learn how to use the tool critically before it can be used effectively and also that difficulties in using the tool effectively may be exposing conceptual difficulties.

Ways of working

There is evidence that students working together in small groups and also working interactively with their teachers in whole classes provided a learning environment in which the ICTs were harnessed effectively. The individual or small group use of the technology gave pupils a valuable opportunity for inquiry and experimentation. However, unless the teacher pulled this together and orchestrated whole class plenaries, each individual student could develop their own idiosyncratic knowledge which might or might not accord with the common knowledge the teacher was intending to develop in the lesson.

There is evidence from one study that students can work with several different ICT tools and evaluate their respective advantages. There is evidence from three studies that students who use ICT out of school were better able to use it effectively within school.

Implications

Limitations

The main limitations of the review are that the constraints involved in terms of time and cost inevitably mean that decisions about the focus of the review question and the review process have to be made to keep the review manageable. This meant we went on to do an in-depth study on two of the areas identified in the systematic map:

- the relationship between different ways of representing functions
- the interpretation of graphical representations of functions

The two other areas have not been subject to in depth analysis are as follows:

- the development of algebraic symbolism
- operations on symbolic expressions

Another limitation of any review of this type is that the individual studies did not set out to answer our review question. They all have different designs and instruments. This is particularly relevant in terms of the tasks used to assess understanding where small differences may make a noticeable difference to the students' responses. Although all the studies in the in-depth review were considered to be evaluations, not all used control groups and not all compared different kinds of software and hardware. So we have evidence of gains but we do not always know if those gains could have been achieved without the use of ICT. Another limitation is the amount of evidence we have of the nature and quality of the teacher input. Most of the studies in this review concentrated on pupils and did not give detailed evidence of how the teachers supported their pupils in developing knowledge of the functional concept and knowledge of how to use the ICT tools. Any conclusions must therefore remain tentative.

Interpretation of the review findings

ICT has a powerful role to play in the development of understanding of functions, but teacher intervention is necessary in helping pupils make meanings and connections between the different representations which it offers.

Application of the review findings

a. Teachers need to help pupils to use the technology critically so that they understand how to interpret the output and in particular how changing scales and windows can change the visual image produced by graphing software. They also need to know how the resolution of the screen image may be constraining and needs to be augmented by alternative information.

b. Teachers need to make links between functions represented symbolically, in tables and in graphs. Symbolic representations give insights into the structure of functions but require some algebraic fluency to produce. Tables of values, whether produced manually or by technology, are an accessible way into the function idea and give an insight into the effect of inputs on outputs. They emphasise a discrete point-wise view of functions, rather than a continuous idea. Graphs produced by technology give a visual image of a function as an object which can be manipulated in its own right but they also give information about particular points on the functions which is of use in solving equations, and in investigating rates of change.

c. Teachers need to negotiate a balance between the individual constructions which may develop when pupils work alone or in small groups with the technology, and common knowledge developed within the whole class. Although this is a consideration in any teaching situation, technology may be particularly fruitful in encouraging individual experimentation. This is desirable but needs to be tempered by teachers encouraging sharing within the whole class. The last point is also relevant when considering the use of electronic whiteboards and computers connected to data projectors. If this is completely within the control of the teacher, then pupils may not have the opportunity to experiment with the technology themselves.

CHAPTER ONE

Background

1.1 Aims and rationale for current review

The Training and Development Agency for Schools (TDA) identified a number of key areas in which systematic reviews of research literature should be carried out over a three-year period from 2003-2006. One of these is the effectiveness of information and communications technology (ICT) in teaching and learning the core curriculum subjects of English, Science and Mathematics. This review focuses on Mathematics.

Although the UK has invested heavily in ICT in schools, it is now clear that simply providing ICT equipment and promoting its use is not enough to produce more than weak gains in attainment. A key finding from one professional user review is that it is the way in which pupils and teachers use ICT that can make a difference (Higgins, 2003). Targeted research-based interventions, which are planned, structured and well integrated, do produce gains in attainment, but even these may not have as much effect as other non-ICT interventions.

In mathematics, despite a considerable literature on ways in which ICT can be used to enhance learning, Ofsted (2004, pp 4-5) reported that 'the use of ICT to promote progress in mathematics remains a relatively weak and underdeveloped aspect of provision…[and] is not as effective as in many other subjects…'. The picture is not entirely negative, however. Sutherland (2004), writing about the InterActive project across subjects and age phases, found that the mathematics teachers in the project had a legacy of ICT use which enabled them to incorporate it more smoothly into their practice and transform their teaching.

One of the ways in which some mathematics teachers have been able to develop this 'legacy of use' has been through reading articles in journals such as Micromath, which has now been amalgamated with Mathematics Teaching, the other journal of the Association of Teachers of Mathematics (ATM). This kind of reporting may be very small scale and localised, but it is accessible to teachers. One of the main aims of this review is to make the best quality evidence available and accessible to teachers, teacher educators and others involved in continued professional development.

Against this background, there were many possible areas in mathematics for the subject of the review. These included focusing on pedagogical issues, specific technologies, software and/or applications, or looking at a specific area of the mathematics curriculum. Given the importance of how teachers use ICT and the decisions involved in terms of choice of technology and software, it seemed important to find evidence of how these factors come together to contribute to teaching in a particular area of mathematics. Algebra is an appropriate focus because it is a crucial aspect for much of secondary phase mathematics, with roots in pre-algebraic activity in the primary phase. In the current version of the Key Stage 3 National Strategy: Framework for Teaching Mathematics, years 7, 8 and 9 (Department for Education and Employment, 2001), there is guidance on the use of ICT and on the teaching of algebra. This review will provide international evidence which may inform future versions of this important policy document for England and Wales.

1.2 Definitional and conceptual issues

Algebra

> *Algebraic symbolism should be introduced from the very beginning in situations in which students can appreciate how empowering symbols can be in expressing generalities and justifications of arithmetical phenomena…in tasks of this nature, manipulations are at the service of structure and meanings. (Arcavi, 1994, p 33)*

This statement highlights the fact that an emphasis on superficial aspects of algebra conceals the true essence of its power. Along with many writers, Arcavi identifies the importance of being able to express **generality** in symbolic terms. This generality may apply to relationships between a variety of mathematical objects, but most pupils will first encounter algebraic ideas in a numerical context. They may first explore ideas of pattern in numbers and express generality in words without recourse to any symbols, but later on they will be introduced to the concise and consistent **symbol system** which gives us the ability to form **expressions** (e.g. **formulae, equations, identities**), which can be used in a variety of problem-solving and reasoning contexts. The National Curriculum makes a distinction between the meaning of these words in terms of the contexts and purposes for which they are used, and the National Strategy suggests that work on relationships between **variables** expressed as formulae, equations, inequalities and identities precedes work on **functions** and **graphs**. It is helpful to think of **functions** to be the overarching concept. Indeed, French (2002, p 3) states that 'one could say that algebra is the study of functions and their application to a wide range of phenomena both within mathematics and from the 'real' world'.

The language and grammar of algebra is not studied for its own sake. It is fundamental to the process of **modelling**, where situations are represented by mathematical models in order to explain, predict, solve problems and prove results. For example, the flow of traffic in a city centre may be modelled by expressing relationships between variables as functions. In the process of trying to solve problems of congestion and keep traffic flowing, equations derived from these functions can be solved to find values of **unknowns**. These values can then be input into devices used to control traffic (e.g. timing in traffic light systems). Modelling involves not only deriving algebraic expressions, but also manipulating and operating upon them.

ICTs

Different ICTs are used to refer to both software and hardware. Within software, the following are included:

- small programs, related to specific aspects of algebra
- programming languages, such as Logo
- spreadsheets and graph-plotting software
- independent / individual learning systems (ILS)
- computer algebra systems

Within hardware, the following are included:

- interactive whiteboards (IWBs) and other projection equipment
- stand-alone computers
- graphical calculators, including those with symbolic capabilities
- Tablet PCs and other personal devices
- Data-loggers

This allows comparisons to be made between the ways in which algebraic ideas are developed using *different software* (e.g. variables using Logo and graph-plotting software). In doing this, there is a need to recognise that the 'algebras' involved in classical algebra, Logo, spreadsheets, graph-plotters and other ICT environments were different from each other, and address questions about transfer between these environments. Comparisons are also made between similar algebraic ideas being developed using *different hardware*: for example, the opportunities offered when graph-plotting is being taught, using devices with the whole class, stand-alone computers or graphical calculators. This provides an opportunity to come to judgments about the relative merits of these different ICTs.

For the purposes of this review, the focus is on learning algebra up to the age of 16, and the use of the internet or videoconferencing are not considered.

Understanding

Understanding is a complex term, but one which is often used in education. It is taken here to be more than the knowledge of definitions or procedures, involving making meaningful connections and relationships with previous knowledge. In algebra, it would involve being able to extend ideas of relationships expressed numerically, and to identify, describe and use generality, functions and graphical representations. Faced with a problem for which algebra could be used, understanding would involve knowing what to do and when to do it, as well as how to do it. The importance of having technical skills is not downplayed, as these skills could be a basis for making connections and are a necessary part of problem-solving. An important part of the Review Group's view of understanding is the ability to operate appropriately in different contexts, and to choose between alternative procedures and representations.

The Review Group does not see understanding as a once and for all state, and would expect pupils to develop more complex webs of connections and representations over time. Since the seeds of algebra may be sown in the primary phase, a lower age limit was not used for the question, but a restriction was made to algebraic ideas involving symbolism.

1.3 Policy and practice background

There is a requirement in the National Curriculum for England (NC) that ICT is incorporated into the teaching of all subjects, and teachers have been required to undertake training under the New Opportunities Fund to improve their ICT competence.

In the NC programmes of study for mathematics (1999), pupils are expected to 'use a variety of resources and materials, including ICT'. The Key Stage 3 National Strategy (Department for Education and Employment, 2001) is explicit both about introducing and developing algebra and the use of ICT. In the strategy, 'ICT includes calculators and extends to the whole range of audiovisual aids, including broadcasts and video film'. Algebra for this age phase is taken to include 'equations, formulae and identities and sequences, functions and graphs', with links made between these topics and with arithmetic. Within the supplement of examples, calculators, spreadsheets, data-loggers, graph-plotters and graphical calculators are all explicitly mentioned.

Despite this inclusion in the written mathematics curriculum, there are still concerns about the use of ICT to promote learning and progress in mathematics. There is an unquestioned assumption (Ofsted, 2004) that ICT is beneficial to learning: 'the most significant impact of ICT is when it is used to enable pupils to model, explore, analyse and refine mathematical ideas and reasoning' (Ofsted, 2004, p 4).

It is also assumed that the problem of ICT use in mathematics teaching is one of implementation. Ofsted argues that there needs to be better distribution of materials, ideas and resources; that schools need better guidance on selecting and using software; and that all schools need to write ICT activities into their schemes of work.

One of the challenges of determining the role of ICTs in the learning of mathematics is that curriculum and mathematical methods may be influenced by the tools available. This is as true for digital technologies today as it was, for instance, when the Greeks used compasses and straight edges in geometry. So, ideas about functions and variables may have subtly different meanings and manifestations within and without an ICT environment. Trying to judge the effectiveness of the ICTs in developing understanding in algebra will have to take this into consideration.

There is also the question of how teachers incorporate ICTs into their existing practices and if they then transform those practices in response to new ways of seeing and doing mathematics. Sutherland (2004) describes how communities of practice - social networks arising out of a desire to teach differently using ICT and to share knowledge, expertise and experience - were created in the InterActive Education Project. The curricular and working context of the studies in this review were examined and included within the synthesis. It is not possible to look at the effectiveness of ICTs without examining the conditions in which they are used.

1.4 Research background

Understanding of algebra

For many pupils, the deeper meanings and purposes for algebra are hidden and they see it as a meaningless activity in which they have to memorise rules and methods for manipulating symbolic expressions (Kieran, 1994). Moreover, although algebra has its roots in arithmetic, pupils often find the transition from the one to the other problematic (Nickson, 2004) as it involves using structural, rather than procedural, features of arithmetic. For this reason, recent work with elementary children in the United States has focused on generalised arithmetic, and has enabled children to progress to the use of algebraic symbolism (Carpenter et al., 2003).

A range of barriers to progress in algebra has been found in the secondary phase (see French, 2002 for a useful summary), including the following:

- Pupils interpreting letters as objects (e.g. a for apples) rather than as unknowns with a specific value or values (e.g. $x + 3 = 10$) or variables which can vary across a range of values (e.g. the x and y variables in the function $y = x - 1$)
- Pupils interpreting expressions simply as processes rather than both processes and objects. For example, pupils may only see $y = x - 1$ as a rule used to draw a straight-line graph, but not also as an object which can be transformed in its own right (e.g. by manipulating constants to produce a set of parallel lines without recalculating values for x and y).
- The isolated practice of skills and routines, which tend to be forgotten
- The lack of meaningful, but not necessarily 'real life' contexts
- The lack of connections between ideas and representations (e.g. between a table of values for a function, its symbolic representation and its graph)

Working with a small group of teachers as part of a larger Teacher Training Agency (TTA) project, Brown (2005, Developing algebraic activity in a 'community of inquirers') helped to develop classroom cultures in which year 7 pupils had a personal need to use algebra. Looking for distinctions – that is, exploring what was the same and what was different in situations - enabled pupils to find structural or algebraic representations useful to them. Teachers also found it helpful for pupils to use writing, both when doing mathematics and also when reflecting on what they had learned. This project also highlighted the advantages of teachers, researchers and teacher researchers working collaboratively.

ICTs and algebra

Much of the background for this section draws on a set of research bibliographies from Micromath (Jones, 2004, 2005a, 2005b, 2005c).

Arguments for the potential of ICT to enhance the teaching of algebra abound, but evidence for its effectiveness is more mixed.

Capponi and Balacheff (1989) found that there was no easy transfer of algebraic knowledge into the spreadsheet context, while Ainley (1996) found evidence that children's understanding of variables was assisted by the use of spreadsheets. More recently, Ainley et al. (2004) operated a spreadsheet-based teaching programme, using the technology as a tool within purposeful tasks. Pupils had a need to use algebraic symbolism and the affordances of the technology stimulated some, but not all pupils, to engage with expressing generality symbolically.

There is some evidence from a case study of Logo use (Harries and Sutherland, 1995) that the computer environment allowed a greater emphasis on the language and structure of algebra, although some difficulties with equivalence and variables were still found.

Much of the research on graphing - which can be done using interactive whiteboards, stand- alone computers or graphics calculators - has tended to focus on graphics calculators. There is evidence from independent experimental studies (e.g. Graham and Thomas, 2000) that 13-14 year-old students using graphics calculators improved their understanding of variables, and that regular users (Ruthven, 1990) employed graphical strategies to solve problems. A review of research published by Texas Instruments (e.g. Burrill, 2002) concluded that the use of graphics calculators helped students improve their understanding of algebra concepts, and encouraged problem-solving in applied contexts and the interpretation of graphs. A subsequent review (Interactive Educational Systems Design, 2003), drawn from the same database but focusing only on those studies with an experimental or quasi-experimental design, found that graphing calculator use led to higher achievement. As well as potential benefits, there is some evidence of difficulties with graphical calculator use. For instance, Wilson and Krapfl (1994) identified problems with scaling, and Mitchelmore and Cavanagh (2000) found that uncritical acceptance of the graphical image on the calculator led students into error.

Interactive whiteboards (IWBs) are a relatively recent introduction to mathematics classrooms in the UK and the research tends to focus on general pedagogical issues, such as pupil participation. Glover and Miller (2001) found that they can be effective, depending on the quality of the teaching, but that the novelty effect of IWBs could wear off. Another study (Godwin and Sutherland, 2004) found the potential for increased understanding of functions and graphs with IWBs within inquiry-based teaching, but also point out the potential of ordinary whiteboards for encouraging interactivity.

Software, such as DERIVE and MAPLE, allows for the symbolic manipulation of algebraic functions and so present similar issues for advanced mathematics as do calculators for arithmetic. This software typically operates on computers (hence the generic name computer algebra systems (CASs)), but more recently, complex calculators, with symbolic as well as numeric and graphical capabilities, have also been introduced. In France, there has been a considerable body of research into teaching and learning mathematics with these tools. Much of this has focused on the complexity of instrumentation, learning to use the new technology so that it becomes a tool for use (Lagrange, 1999). Using CAS places technical and conceptual demands on students as they require mastery of the formal ways of interacting with the software, and the ability to interpret the results of operations (Artigue, 2002). This requires time and carefully designed activities. Ruthven and Hennessy (2002) point out the difficulty of fully realising the potential of CAS if they are not given status within secondary school mathematics.

ICT use in context

The quality of teaching has already been mentioned in the context of IWBs. In the spreadsheet context, Rojano (1996) found evidence that judicious use of spreadsheets led to algebraic understanding. A review of graphic calculator use (Penglase and Arnold, 1996) warns that many research studies do not clarify the relationship between the use of the graphic calculator and the context in which it is being used. Rodd and Monaghan (2002) found a range of teacher factors in determining graphical calculator use, including their positive regard for calculators as a learning aid and their perceptions that computers were a higher resource priority. Teachers clearly mediate the use of ICT in their classrooms and have views on the features of successful ICT use, together with concerns and qualifications (Ruthven and Hennessy, 2002). These views and constraints will clearly affect how teachers integrate ICT into their teaching. As well as the salience of the nature of tasks and the role, knowledge and beliefs of the teacher, Doerr and Zangor (2000) found that student communication was sometimes inhibited by the use of the graphics calculator as a personal device, but that, when shared, whole class learning was supported.

The review therefore aims to clarify the conditions under which ICTs can be used to develop understanding of algebraic ideas. From the above, it is clear that the teacher's role is crucial. Some evidence was found about the ways in which the teachers worked with the technology, the tasks they used, the pedagogical practices they adopted, and, in some cases, how they used different technologies in complementary ways. This last issue is important in the context of England and Wales as electronic whiteboards become much more common.

1.5 Authors, funders and other users of the review

The Review Group consists of key groups involved in mathematics education from universities, schools and local education authorities. All have a professional interest in both the substance of the review and the methodological approach to systematic reviewing. For this review, the existing EPPI-Centre Review Group for Mathematics was enlarged to include members with particular expertise in ICTs and Mathematics. This review was led by Maria Goulding, who has worked as co-investigator with Chris Kyriacou on two previous EPPI-Centre reviews. They have both have published substantive and methodologically focused papers in academic journals based on previous reviews, and both are involved in the professional preparation of secondary mathematics teachers, for whom the outcomes of this review are particularly important.

The project has been funded by the Training and Development Agency for Schools (TTA), which is concerned with bringing reviews of research literature to bear on the training and continued professional development of teachers. It is hoped that the results of this review will inform beginning and continuing teachers about the impact of ICT on a crucial aspect of the mathematics curriculum. The review not only identified studies in which the use of ICT was shown to be effective in the teaching of algebra, but also the conditions under which this effectiveness occurred.

As well as those involved in mathematics curriculum research and policy-making, the principal audiences for the review are likely to be teacher educators, researchers and policy-makers involved in the initial and continuing preparation of mathematics teachers. The recent setting up of the National Centre for Excellence in Teaching Mathematics (NCETM), and the appointment of regional advisers provides a forum and mechanism for dissemination, as well as the existing academic and professional networks and conferences.

As with previous mathematics reviews, dissemination will take place through internet access to the review report, conference papers and publication in refereed journals. Conference presentation planned for 2007 are at a one-day conference for the British Society for Learning Mathematics and the annual conference of the British Educational Research Association.

1.6 Review questions

The review group agreed to the initial question:

How have different information and communication technologies (ICTs) contributed to the development of understanding of algebra for pupils up to the age of 16?

After finding and categorising studies which addressed this question the question was narrowed down (see section 2.3.1) to the following:

How have different information and communication technologies (ICTs) contributed to the development of understanding of functions for pupils up to the age of 16 (with particular reference to the relationships between different representations and the interpretation of graphical representations)?

CHAPTER TWO

Methods used in the Review

2.1 User involvement

2.1.1 Approach and rationale

Initial discussions for this review have been held with the TDA, the English and Science review teams at York, the Mathematics Review Group and other teachers in schools not in the Review Group. The Mathematics Review group - which includes teachers, teacher trainers, educational researchers and a local education authority adviser - met and discussed several possible foci before deciding on the question of the review. The experiences of trainee teachers in schools have also been taken into account, following discussions after observed lessons.

These groups represent the main users of the review and were consulted at later stages. The Mathematics Review Group met twice during the progress of the review: once to decide on the research question and once during the key-wording process. The British Society for Research into the Teaching and Learning of Mathematics (BSRLM) was also consulted and emergent findings will be presented at a one-day conference, as for previous Mathematics Education EPPI-Centre reviews. When the final review has been approved by peer referees, it will go out to three users to provide user perspectives which can be published on REEL alongside the final review (as for the first EPPI-Centre Mathematics Education review).

2.1.2 Methods used

Systematic review methods, using the EPPI-Centre guidelines and tools for conducting systematic reviews, were used.

2.2 Identifying and describing studies

2.2.1 Defining relevant studies: inclusion and exclusion criteria

Papers included in the systematic map reported studies on the effectiveness of different ICTs on the development of understanding in algebra for pupils up to the age of 16. As the focus of the study is on the effects of ICT, papers using methods to identify such effects are required. Thus focus is on evaluations, either naturally occurring or researcher manipulated.

The review is limited to the period between 1996 and 2006. This is quite a generous timeframe, given rapid developments in the field.

Inclusion criteria

- Must be an empirical study of the effects of ICTs, as defined for this review, in mathematics teaching
- Must be a study of the effects of using different ICTs, as defined for this review, on understanding in algebra, as defined for this review
- Must focus on students up to the age of 16
- Must be in a mainstream school setting
- Must be an evaluation study
- Must be in English and published in a professional or academic journal, or presented at an academic conference between 1996 and 2006

Exclusion criteria

EXCLUSION ON SCOPE

A study will be excluded if it is:

1. Not an empirical study of ICTs used in teaching/ learning of mathematics (For example, studies of ICT used only in assessment of mathematics are excluded.)

2. Not focusing on the specified ICTs (i.e. ICTs included are small programs, programming languages, such as Logo, spreadsheets and graph plotting software, computer algebra systems, ILS (independent/individual learning systems), interactive whiteboards (IWBs) and other projection equipment, stand-alone computers, graphical calculators, data- loggers but not internet, videoconferencing, broadcast and video film)

3. Not focusing on the effects on understanding algebra as defined for this review (For example, probability, statistics and calculus are not included, even though algebra may be used in these areas.)

4. Not focusing on children or young people up to the age of 16

EXCLUSION ON STUDY TYPE

A study will be excluded if it is:

5. A study categorised according to the EPPI-Centre current classification as:

A description

B exploration of relationships

D methodology

E review

or a collection of articles. (Some databases have single entries for collections or conference papers.)

EXCLUSION ON DATE AND TYPE OF PUBLICATION / SOURCE

6. Not published in an academic journal or presented at an academic conference between 1996 and 2006.

2.2.2 Identification of potential studies: search strategy

Papers were identified from the following sources:

- Searching the electronic bibliographic databases: Educational Resources Information Center (ERIC), British Educational Index (BEI), Australian Education Index (AEI)

- Handsearching proceedings of recent conferences and handbooks of the British Society for Research into Learning Mathematics (BSRLM), the International Group for the Psychology of Mathematics Education (PME), the International Conference on Technology in Mathematics Teaching (ICTMT). This identified studies which were too recent to have been published in academic journals.

- Handsearching key academic and professional journals:

  - *Educational Studies in Mathematics*
  - *International Journal for Technology in Mathematics Education (formerly International Journal for Computer Algebra in Mathematics Education)*
  - *International Journal of Computers for Mathematics Learning*
  - *Journal of Computer Assisted Learning*
  - *Journal of Mathematical Behaviour*
  - *Journal for Research in Mathematics Education*
  - *For the Learning of Mathematics*
  - *Mathematics Teaching*
  - *Mathematics in Schools*
  - *Micromath*

A check was also be made by asking for recommendations from contacts with expertise in this area. Higher degree theses were not included as it was not possible to access these systematically for countries outside the UK. However, conference papers from such work should be identified from handsearching.

Keywords and descriptors include the following:

ICT (and variations, such as information and communication technology, etc.)

Algebra

Logo

Spreadsheets

Graphing (and variations such as graphics calculators, etc.)

Integrated learning systems

Interactive whiteboards

Data logging

Computer algebra systems

The What Works Clearing (WWC) House reviews were not searched or screened for the review.

2.2.3 Screening studies: applying inclusion and exclusion criteria

The Review Group set up a database system, using EndNote bibliographic software, for keeping track of and coding studies found during the review. Inclusion and exclusion criteria were applied successively to (i) titles, (ii) abstracts and (iii) full papers. Full papers were obtained for those studies that appeared to meet the criteria or where there was insufficient evidence to be sure. The inclusion and exclusion criteria were re-applied to the full papers and those that did not meet the initial criteria were be excluded. The database was fully annotated with reviewer decisions on inclusion and exclusion and reasons for exclusion.

2.2.4 Characterising included studies

The studies remaining after screening were keyworded, using the current EPPI-Centre Core Keywording Strategy (version 0.9.7) and online software EPPI-reviewer. Additional keywords, specific to the review, were added. All the keyworded studies were uploaded to the larger EPPI-Centre database, Research Evidence in Education Library (REEL), for others to access via the website.

2.2.5 Identifying and describing studies: quality-assurance process

Application of the inclusion and exclusion criteria, and the keywording was conducted by pairs of the Review Group working independently and then comparing their decisions and coming to an agreement. Members of the EPPI-Centre assisted in applying criteria and keywording studies for a sample of studies.

2.3 In-depth review

2.3.1 Moving from broad characterisation (mapping) to in-depth review

After mapping all the included studies, they were categorised as focusing on:

1. the development of algebraic symbolism
2. multi-representations of functions
3. graphical representations of functions
4. operations on symbolic expressions

Most studies could be placed into one or more of these categories, but, in two studies, it was not clear which aspect of algebra was being addressed by the ICTs. The research question and the inclusion/exclusion criteria were narrowed and refined for the in-depth review. The narrowed research question was as follows:

> *How have different information and communication technologies (ICTs) contributed to the development of understanding of functions for pupils up to the age of 16 (with particular reference to the relationships between different representations and the interpretation of graphical representations)?*

For a study to be included in the in-depth review, it made explicit which aspect of algebra was being addressed with the ICTs and had a focus on the multi-representation of functions, including graphs.

Exclusion on scope

A study was excluded at this stage on the following grounds:

1. It did not make explicit which aspect of algebra was being addressed by the ICTs.
2. It did not address the multi-representations of functions including graphs.

2.3.2 Detailed description of studies in the in-depth review

Studies identified as meeting the inclusion criteria were analysed in-depth, using the EPPI-Centre's detailed Data-extraction Guidelines (EPPI-Centre, 2002a) together with its online software, EPPI-Reviewer (EPPI-Centre, 2002b).

2.3.3 Assessing quality of studies and weight of evidence (WoE) for the review question

The following three components were used to give different quality ratings to the findings and conclusions of the studies in the in-depth review:

A. Soundness of studies (internal methodological coherence) based on the study only (WoE A)

B. Appropriateness of the research design and analysis used for answering the review question (WoE B)

C. Relevance of the study topic focus to the review question (WoE C)

Each of these three components was assessed as low, medium or high (scored 1 to 3 respectively). Studies were judged to provide a high WoE on A and B, if the analysis was deemed to be transparent and appropriate to the research method. Studies were judged to give high WoE on C, if they provided good detail of the ways in which pupils and/or teachers were working.

In considering the overall WoE D, priority was given to considerations of:

- *Relevance (WoE C) to the review question.* This was in line with the research question which sought to determine how the technology contributed to the understanding of functions. The Review Group was looking for the best available evidence to support the researchers' claims and to answer the review question. This did mean that some studies reporting the work of small numbers of pupils were still considered to have an overall high weight of evidence. In some cases where there were some methodological shortcomings, it was still judged that the study had made a significant contribution to addressing the review question.

2.3.4 Synthesis of evidence

The data was synthesised to bring together the studies which provided the best available evidence for answering the review question. The synthesis summarised the effects of ICTs on the understanding of algebra, and the contexts in which those effects were achieved.

2.3.5 In-depth review: quality- assurance process

Data-extraction and assessment of the weight of evidence brought by the study to address the review question was conducted by pairs of RG members, working independently and then comparing their decisions and coming to a consensus. Members of the EPPI-Centre also assisted in applying criteria for data extraction and quality assurance for a sample of studies.

CHAPTER THREE

Identifying and describing studies: results

3.1 Studies included from searching and screening (see Figure 3.1)

The electronic search and citation searching identified 625 papers, using the specified search strategy, and 18 duplicates were excluded. In the first stage of screening on titles and abstracts, the six exclusion codes were applied to each of these by a member of the Review Group, resulting in 454 exclusions. The majority of these papers were excluded using exclusion code 1; that is, they did not report empirical study of ICTs used in teaching/learning of mathematics.

Full copies of the remaining 153 papers were then screened, using the inclusion/exclusion criteria. In addition a further four papers were identified as a result of expert contact and were added to the main review database.

The six exclusion codes were then applied to a full length copy of 144 of these 157 papers; a full length copy of 13 of these papers was not available. This resulted in a further 110 papers being excluded. Here the most common criteria for exclusion were exclusion code 1 as above, exclusion code 3 (not focusing on algebra as defined for this review) and exclusion code 5 (study type).

This resulted in 34 papers reporting 33 studies being identified for the systematic map.

3.2 Characteristics of the included studies (systematic map)

The EPPI-Centre keywording strategy and review-specific keywords (Appendix 2.4) were applied to the 33 papers in order to develop the systematic map. Of these, 21 had been identified by electronic screening, eight by citation searching and four by expert contact.

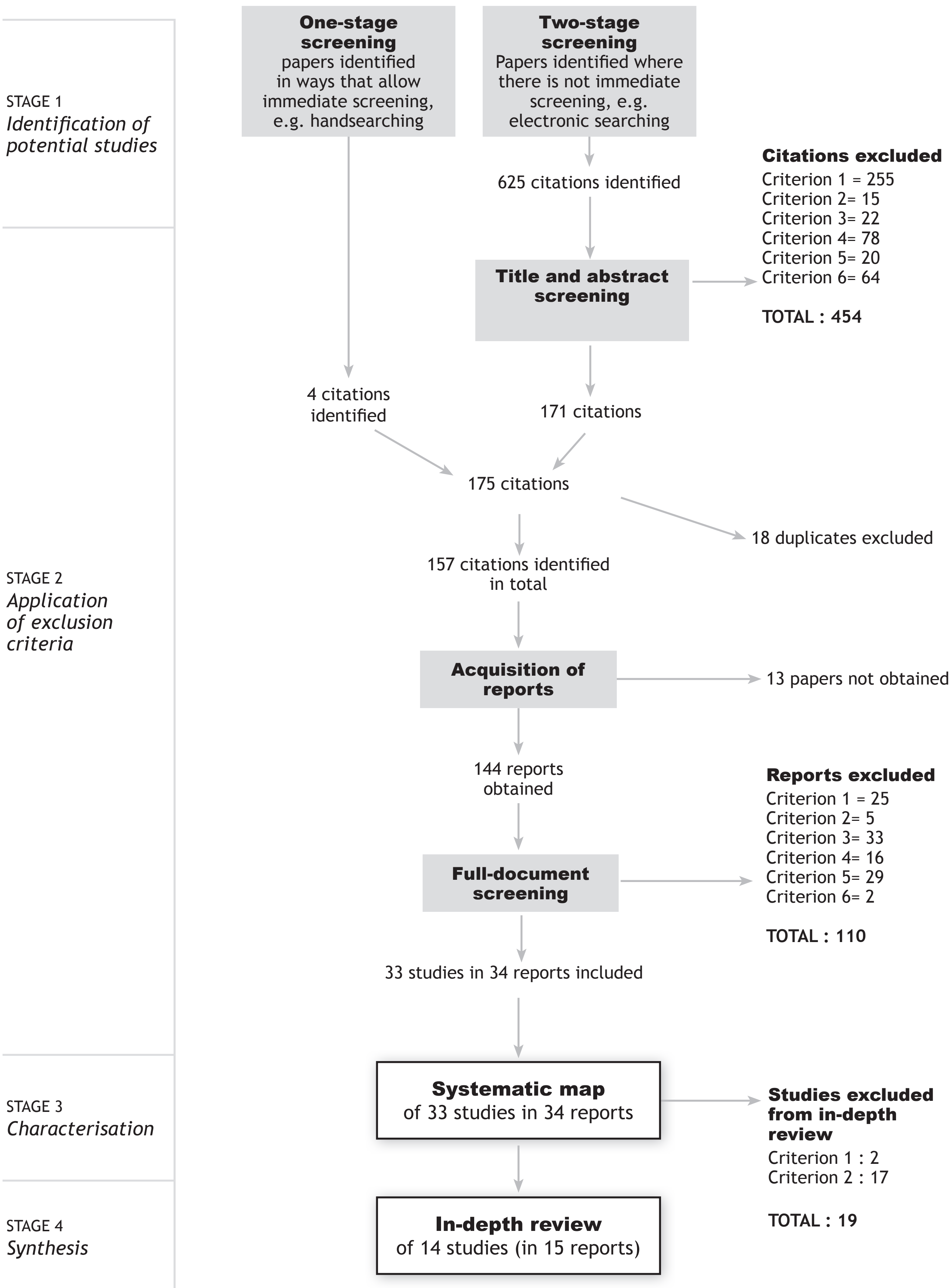
Nineteen of the papers were published in refereed journals, seven were in refereed conference proceedings, four in conference proceedings, one in a refereed collection of conference papers, one was a research report and one was a refereed conference paper. So the majority of the papers were in high quality research journals and all but five of the papers had been peer-reviewed.

All the studies were considered to be evaluations except for one descriptive study (Wilson and Ainley, 2006) which amplified another study and was directly related to the review question. Fifteen of the studies were naturalistic evaluations (Bills et al., 2005; Borba and Confrey, 1996; Cedillo, 2001; Clark and Redden, 2000; Doerr and Zangor, 2000; Drijvers, 2004; Friedlander and Stein, 2001; Gage, 2002; Godwin and Beswetherick, 2002; Godwin and Sutherland, 2004; Gomes-Ferreira, 1998; Gray and Thomas, 2001; Healy and Hoyles, 1996; Hershkowitz and Kieran, 2001; Yerushalmy, 2000) and 17 were researcher-manipulated (Aczel, 1998; Carter and Smith, 2001; Drijvers and van Herwaarden 2001; Graham and Thomas, 2000; Hegedus and Kaput, 2003; Isiksal and Askar, 2005; Kramarski and Hirsch, 2003; Merriweather and Tharp, 1999; Mitchelmore and Cavanagh, 2000; Morgan and Ritter, 2002; Ninness et al., 2005; Norton and Cooper, 2001, Norton et al., 2002, Sivasubramaniam, 2000; Strickland and al-Jumeily, 1999; Tynan and Asp, 1998; Zehavi, 1997). Some were small scale and some larger scale, and some included a mixture of quantitative and qualitative data.

All the studies were written in English. Nine studies were conducted in England, seven in the USA, five in Australia, four in Israel, two in New Zealand, two in the Netherlands, and one each in Mexico, Brazil, Canada and Turkey.

Thirty-two studies had a population focus on pupils in the 11-16 age range. Three of these also focused on teachers (Doerr and Zangor, 2000; Godwin and Sutherland, 2004) and one focused on teachers only (Wilson and Ainley, 2006).

Figure 3.1 Filtering of papers from searching to map to synthesis



3.3 Identifying and describing studies: quality-assurance results

Quality assurance at the first stage (title and abstracts) of screening

A sample of the 607 citations identified at the first stage of screening was screened by a second member of the Review Group and a random sample of 20 citations was then screened by a member of the EPPI-Centre in London. The small number of disagreements were mainly due to interpretation of technical mathematical terms and were moderated satisfactorily.

Quality assurance at the second stage (full papers) of screening

A sample of the 144 full reports was screened by a second member of the Review Group. At the Review Group meeting on 26 July 2006, 10 members of the Group looked closely at five studies in order to moderate judgements.

Quality assurance for keywording

After full document screening, the 33 studies were keyworded electronically, using the EPPI-Centre keywording strategy for classifying educational research and using the Mathematics ICT review-specific keywords. Of these studies, 12 were keyworded by two members of the Review Group or EPPI-Centre team, and one was keyworded by three members.

Quality assurance for data-extraction for the in-depth review

Data from the 14 studies chosen for the in depth review was extracted electronically, using the guidelines for extracting data and quality assuring primary studies in educational research (version 0.9.7) and review-specific data extraction. All 14 were data-extracted by two members of the Review Group or EPPI-Centre team, and one was data-extracted by three members.

3.4 Summary of results of map

The 33 studies in the systematic map were categorised using the review-specific keywords questions 1 and 3.

Ten studies addressed the development of symbolism: Clark and Redden, 2000; Drijvers 2004; Gage, 2002; Graham and Thomas, 2000; Healy and Hoyles, 1996; Hegedus and Kaput, 2003; Hershkowitz and Kieran, 2001; Tynan and Asp, 1998; Wilson and Ainley, 2006; and Yerushalmy, 2000.

Eight studies addressed the relationship between different ways of representing functions: Doerr and Zangor, 2000; Friedlander and Stein, 2001; Gomes-Ferreira, 1998; Godwin and Beswetherick, 2002; Gray and Thomas, 2001; Mitchelmore and Cavanagh, 2000; Ninness et al., 2005; and Yerushalmy, 2000.

Fourteen studies address the interpretation of graphical representations of functions: Borba and Confrey, 1996; Doerr and Zangor, 2000; Friedlander and Stein, 2001; Gomes-Ferreira, 1998; Godwin and Beswetherick, 2002; Godwin and Sutherland, 2004; Gray and Thomas, 2001; Hegedus and Kaput, 2003; Hershkowitz and Kieran, 2001; Isiksal and Askar, 2005; Mitchelmore and Cavanagh, 2000; Ninness et al., 2005; Sivasubramaniam, 2000; and Yerushalmy, 2000.

Eighteen studies addressed operations on symbolic expressions: Aczel, 1998; Bills et al., 2005; Cedillo, 2001; Doerr and Zangor, 2000; Drijvers, 2004; Drijvers and van Herwaarden, 2001; Friedlander and Stein, 2001; Gray and Thomas, 2001; Hershkowitz and Kieran, 2001; Isiksal and Askar, 2005; Kramarski and Hirsch, 2003; Merriweather and Tharp, 1999; Norton and Cooper, 2001; Norton, Cooper and McRobbie, 2002; Strickland and Al-Jumeily, 1999; Tynan and Asp, 1998; Yerushalmy, 2000; and Zehavi, 1997.

Two studies did not report what the ICT tool does and did not give any details of activities undertaken by students: Carter and Smith, 2001; and Morgan and Ritter, 2002.

CHAPTER FOUR

In-depth review: results

4.1 Selecting studies for the in-depth review

For a study to be included in the in-depth review, it had to make explicit which aspect of algebra was being addressed with the ICTs and this had to have to focus on the different ways of representing functions, including the interpretation of graphical representations.

Exclusion on scope

A study was excluded at this stage if:

1. it did not make explicit which aspect of algebra was being addressed by the ICTs;
2. it did not address the different ways of representing functions including the interpretation of graphical representations.

4.2 Comparing the studies selected for in-depth review with the total studies in the systematic map

All 14 studies which addressed the graphical representation of functions were selected for the in-depth review. This included all eight studies which addressed the different ways of representing functions. The in-depth review excluded nine of the ten studies addressing the development of algebraic symbolism, and 12 of the 18 studies which addressed operations on symbolic operations. The in-depth review excluded the two studies which did not give sufficient detail of the software or the activities.

4.3 Further details of studies included in the in-depth review

Borba and Confrey (1996) questioned the traditional approach to the transformation of functions, which moves from symbolism to graphs. Their study investigated the feasibility of inverting this sequence by going from graphs to tables of values to symbolism, using the Function Probe software. They hypothesised that an emphasis on visualisation would allow students to move more easily into algebraic symbolism, while maintaining visual meaning for the symbolism.

This was a case study of one 16 year-old student, Ron, from an alternative community school in the USA working with a researcher who conducted clinical interviews over a period of five weeks. The focus was on how this student made links between transformations first made by directly operating on the visual image and changes in the coordinates in tables of values, then finally with changes in the symbolic form of the transformed function.

The student was introduced to the software and followed a sequence of tasks involving transformation of functions, starting with visual transformations. He was asked to predict the effect of transformations on tabular forms and then on symbolic forms. The student's and researchers' words and code were captured using video and audio tape.

Ron was able to make meaningful connections between the visual, tabular and symbolic representation. He was able to resolve the differences between his predictions and the computer feedback by introducing a mediating metaphor, that of a double rubber sheet and thread. Clearly this was a very particular, resource-intensive intervention with an individual student, which also depended on the multi-representational facilities of the computer software. The authors conclude that this approach has potential, but much depends on the careful construction of tasks and the opportunity for listening closely to students and guiding their thinking.

The study was assessed as having medium weight of evidence.

Doerr and Zangor (2000) focused on the co-ordination of psychological aspects of learning within the social context of the classroom with particular reference to tools and norms. Their questions were as follows:

- How does the teacher's knowledge, beliefs and role affect the use of the graphics calculator (GC) in the classroom?
- How do students use GCs when learning mathematics?
- How does the teacher's role, knowledge and beliefs interact and relate to the student's GC use?
- What are the constraints of GC use?

This was a US classroom-based observational case study of one experienced teacher with two pre-calculus classes, one of 17 students and one of 14 students, aged between 15 and 17. The classes were observed over three units of study covering linear, exponential and trigonometrical functions over a period of 21 weeks. The students all had either TI 82 or TI 83 calculators and had used them for a year previously. The classroom data was collected using video and audio tape; the teacher was interviewed and involved in corroborating the data analysis.

Students used the tool in five ways:

- as a computational device
- as a transformational tool
- for data collection and analysis
- as a visualising tool
- for checking

They were encouraged to work meaningfully, to be alert to the limitations of the tool and methods (e.g. relying on appearance of graph, relying on regression tools for curve fitting without mathematical justification) and to share their thinking in whole class discussion. They were often invited to take the lead in addressing the whole class. As time went on, when in small groups, pupils tended to work individually, but when in whole class discussion, the sharing supported whole class understandings.

The results showed that the role, knowledge and beliefs of the teacher and the nature of the tasks resulted in the emergence of a rich use of the graphics calculator. The teacher created a rich learning environment in which the students were able to use the GC critically, so that the calculator did not become the mathematical authority. On a cautionary note, the authors conclude that graphics calculator use could lead to very individual constructions and pathways without the use of whole class sessions where students shared their thinking.

The study was assessed as having high weight of evidence.

Friedlander and Stein (2001) reported from Israel on an aspect of the Compumath project, a junior high school curriculum within an interactive computerised environment. They set out to investigate the following:

1. students' solutions to equations when they have a choice of tools (paper and pencil and electronic tools)
2. students' ability to choose, use and integrate various representations
3. students' views on the tools

Students were asked to find solutions for a linear equation with decimal coefficients, a quadratic equation with integer roots, a pair of linear equations, and a pair of equations (one linear and one quadratic). They could use paper and pencil methods, spreadsheets, graph-plotting software and an algebraic symbol manipulator (Derive).

The study focused on six pairs of 13-14 year-old higher and average ability students from a selective school, using observational data and the students' written solutions. The solutions for pairs were classified by tool and type of approach, and interviews for individuals were used to identify preferences.

The students used an average of 3.8 methods per pair and used all four available tools. They had a low level of success in solving the quadratic equation using paper and pencil, but they had not been taught the algebraic formula before. There was a low frequency of spreadsheet use, but they had not been taught explicitly on the course how to solve equations using spreadsheets. All pairs found the solution to each equation or pair of equations. Students tended to start using paper and pencil, and then moved to using a computer tool. They preferred the paper and pencil method for the linear equation, but the computer tools for the quadratic and simultaneous equations. Students thought that the symbolic manipulator was quick and easy, but that it did not help them understand the solution process; they also felt that the advantage of the graph-plotter was its transparency.

The students were able to present the equations in various representations, move between tools and between representations, and connect the outcomes, although they had not been taught to do this before.

This study was assessed as having medium weight of evidence.

As part of the ESRC funded InterActive Education: teaching and learning in the Information Age project, *Godwin and Beswetherick (2002*) focused on the learning and understanding of quadratic functions, using the graphical software package, Omnigraph, in an English secondary school.

This study was informed by a theory of software packages as 'learning environments' or microworlds, which give learners an opportunity to experiment with mathematical ideas.

The study aimed to find how one female year 9 student interacted with the software within the teaching programme designed by her teacher in conjunction with the researchers, what she learned, and how she experimented. It investigated the role of task structure in directing prescribed and experimental work

From six students, closely observed in class and interviewed, this one girl was chosen for the case study. It used a rich data set of pre-intervention assessment, post-intervention assessment, pre- and post-intervention interviews, observations of interactions and responses in whole class sessions, and a collection of student's written response to tasks. A digital video camera and minidisk recorder focused on the student during all individual work.

Kay's graphical output was related to her written work on the worksheets and coded according to whether the graphs were (i) anchor graphs (prototypes), (ii) prescribed graphs - graphs of functions given by the teacher or (iii) experimental graphs in response to more open-ended questions where the effect of changing coefficients and constants is required. Series of related experimental graphs were coded as 'runs'.

Kay was unable to plot any of the three graphs in the pre-intervention assessments, but she was able to sketch them afterwards. She demonstrated a good understanding of the behaviour of quadratic graphs under varying conditions. She was able to reproduce two graphs from sketches without any help, and two with some minor researcher input. The authors also report that Kay had enjoyed the work and that the tasks encouraged all the students to reflect on their actions, and to think and predict, thereby enabling them to gain conceptual insights.

The authors conclude that creating the right learning environment so that a relatively easy to use software like Omnigraph can result in successful learning requires planning and thought. This design used whole class engagement at the beginning of the lesson where the teacher used the software with a data projector and an ordinary whiteboard, the students then worked on computers and finally the teacher drew them all back together for a plenary. This combination, together with the range of closed and more open tasks on the worksheet, allowed a mixture of prescription and experimentation. In this way, students kept to task but had a rich learning experience. The teacher felt he could open up some of his tasks more in future, having looked at the video data.

This study was assessed as having high weight of evidence.

Godwin and Sutherland (2004) report on a study, also part of the InterActive Education project, which aimed to find out how teachers and pupils use digital tools for enhancing the learning of functions and graphs, and how they interact with each other in the classroom. It was framed by a sociocultural perspective on learning with specific attention to:

- tool use (digital, non-digital and the 'master' tool of language)
- knowledge building in communities

The study focused on a sequence of four lessons of two different classes of 13-14 year-olds taught by two different teachers, Rachel and Rob, in two English schools.

The different learning activities designed and used by teachers to teach pupils about functions used digital and non-digital technologies, and involved some whole class and some individual and pair work. Rachel's students used graphics calculators and she worked with a non-digital whiteboard and overhead projector with transparencies operated dynamically. Rob's students operated Omnigraph on computers in a computer suite and Rob worked with a digital projector for the whole class.

Pupils were diagnostically assessed before and after the intervention, and were interviewed after the teaching to assess understanding. Videotape captured observational data of pupils and teachers during the lessons, including the teachers' board work. The analysis consisted of marking the assessments, viewing and re-viewing the videos, transcribing the lessons, and coding the transcripts, using multiple perspectives (teacher talk, teacher space, pupil space, tool use, ways of working, nature of mathematical learning).

Both classes improved their understanding of functions during the course of the lessons. Two relatively low achieving boys, focused on in the analysis of Rachel's third lesson, experimented with their own ideas as well as Rachel's and made considerable gains. The calculator was a tool which enabled them to investigate in ways that would have been difficult with paper-based technology. Some students, however, were less engaged with the mathematical activity. In Rob's class, one of the girls (the same girl reported in the previous paper), focused upon in the analysis, demonstrated understanding of how the parameters in the quadratic functions affected the representation and also how changes in the scale affected the appearance of the function. She therefore went beyond the artefacts of the medium to learn about the attributes of the quadratic functions.

The two teachers worked in different ways, but both used prototype functions. Pupils within the same class experimented with the ICTs in multifarious ways. When pupils worked individually with the calculators, it was difficult to see what was on their screens but there was collaborative mathematical talk. When one mixed pair in Rachel's class shared the calculator, the boy took possession of the calculator and their talk was social, not task related. Pupils worked individually on the computers in Rob's class, but there was often communication of ideas across the classroom.

In terms of the building of a knowledge community, Rachel used the pupils' responses in whole class interactions to build shared knowledge, using the whiteboard and OHP. Rob used projected computer images in whole class plenaries and encouraged pupils to come to the front of the class to make predictions, which were then tested.

The authors conclude that there is a tension between pupils experimenting with the technology and hence developing individual and idiosyncratic knowledge, and the development of collective mathematical knowledge in the community of the classroom. Teachers have a considerable amount of choice in the design of teaching sequences, the way in which they use the technology with pupils, and the way in which they interact with the class. Effective tools include non-digital technologies, such as the ordinary whiteboard, as well as digital technologies.

This study was assessed as having high weight of evidence.

Gomes-Ferreira (1998) examined the following:

- the different ways students interacted in three computer microworlds designed to explore functions
- the ways in which students construct conceptions of functions in these different computer microworlds
- the linkage of conceptual ideas across these environments

This was a longitudinal study of four pairs of Brazilian students over 13 meetings of two hours each. Assessment data from a pre-test and post-intervention interview data were analysed.

The Dynagraph (DG) microworld had two different modes: Parallel and Cartesian. The DG Parallel represents a function point by point using two sprites (one representing x and the other f(x)) so that the motion of the two sprites is observed giving a co-variational representation. The Cartesian version is similar, but includes axes and a dot representing (x, y). Function Probe is a software tool which combines algebraic, graphical and tabular representations.

The authors claim that in the DG Parallel microworld students demonstrated a co-variational concept of function, but line symmetry and periodicity were rarely identified. In the DG Cartesian microworld, they also developed a co-variational view. There were few attempts to link the representations in the DG Parallel microworld with existing school knowledge of functions. This was a new representation where students seemed free of previous conceptions. When, however, they did make the connections between the behaviour of the sprites and whether the function was increasing or decreasing, as one pair did in the final interview, these were robust. When using DG Cartesian and Function Probe, they did make connections with terms from their school mathematics. With Function Probe, the students tended to explore and modify their existing conceptions, and made connections between algebraic and Cartesian representation. These results seem to suggest that the students did make different links with their conceptual knowledge of functions within the three microworlds, but that each offered them different opportunities. It is difficult to infer whether the authors are advocating the use of all three microworlds in furthering different aspects of the students' conceptual understanding or whether they see one microworld as giving greater opportunities.

This study was assessed as having low weight of evidence.

The New Zealand study by Gray and Thomas (2001) explored students' understanding of the relationship between the graphical, symbolic and tabular representation of the quadratic function and in particular how these relate when solving quadratic equations.

The research questions were as follows:

1. What is the students' ability to work within the symbolic representation (e.g. when linear and constant terms are added to both sides of an equation)?
2. Could they solve an equation presented in one representation by using another?
3. Could they relate processes for solving equations in tabular, symbolic and graphical representations?

A mixed sex group of 25 14-15 year-old students from a second stream of a private school in Auckland, who were new to the use of graphics calculators, were initially taught how to expand and factorise quadratic expressions, solve quadratic equations and graph quadratic functions by plotting graphs by hand. They then took part in a module of three fifty-minute teaching lessons, introducing them to the multi-representational facilities of the calculator which were then used to solve quadratic equations, using an inter-representational approach. Students worked from a booklet and were encouraged to work in small groups; the teacher and researcher taught the class and circulated giving help.

Data from parallel pre- and post-test, with the post-test also used as a delayed post-test, was used to gauge students' equation solving skills. The 17 students who completed all the tests did better on the post-test and there was some evidence of some sustained improvement. Five students scored higher on the delayed post-test than on the post-test. There was no change in solution rate for the symbolically presented equations, and in particular there was poor understanding of the balance model of equation. Many students did not make connection between the different representations, with a reliance on the symbolic form and a pointwise process perspective of function. There was poor use of tables for solving equations. However, at least two students did relate the different representations.

Despite improvements in test scores, the authors conclude that the use of the graphics calculator was not as successful at building up representational fluency as hoped and that the students did not improve their understanding of some aspects of quadratic functions. Their interactions with each representation was largely process rather than object-oriented: that is, they saw the representation as something to be produced, rather than something that could be worked upon in its own right. Their progress may have been impaired by not being able to take the graphics calculators home. The authors remain convinced of the potential of graphics calculators, but feel they have not yet found the right pedagogical format.

This study was assessed as having high weight of evidence.

Hegedus and Kaput (2003) assessed the effectiveness of a system where pupils worked in pairs on computers and passed their individual work to the teacher, who aggregated and projected it for the whole class onto a whiteboard display. The material addressed the algebra of change and variation. The researchers were particularly interested in how classroom connectivity could enable new and intense forms of social interactions.

Twenty-five middle and high school students from a sample of 38 completed an after- school algebra enrichment course over five weeks. The middle school students were judged to be higher achievers and the high school students were lower achievers, based on state exam scores. The evaluation used a 20-item pre-post test design, using questions drawn from several sources, including USA state examination questions.

The scores from the 24 students who completed both tests are used in the analysis. This gave outcome measures for the two groups, and Item by item analysis gave information of performance on specific areas.

The results show that the connected SimCalc classroom had a significant effect on the students' learning. The effect size for both groups of students was extremely high (1.78, 1.91) and the ninth grade students had a greater effect size (1.91). The seventh and eighth graders had a greater gain relative to their performance on the pre-test. Statistical checks showed that the gain was mainly based on the intervention, and was not related to previous knowledge. Item by item, there were eight statistically significant gains involving interpreting graphs, interpreting $y = mx + b$ as m and b vary, linearity, interpreting slope in real life situations, generating and interpreting families of functions. Although the intervention had concentrated on linear functions, the students made most gains on an item requiring interpretation of non-standard algebraic relationships of two geometric quantities.

The authors conclude that, by combining the dynamic SimCalc environment with classroom connectivity, students' performance on tenth grade algebra related questions can be improved in a short period of time.

This study was assessed as having high weight of evidence.

Hershkowitz and Keiran (2001) set out to investigate how students used graphics calculators to solve a problem, and how they interweaved working in a mechanistic algorithmic way with working in a meaningful way.

The activity, an investigation of growth patterns, had previously been used with an Israeli group and was now presented to a Canadian class working in groups of three. One group is the focus of this research. Data on pupils' speech, written work, calculator key presses and screen images was collected.

The Canadian group, using TI 83 + calculators, first worked in a mechanical fashion, using the linear regression facility. They had been exposed to this facility in previous work where curve fitting was used to help model real life data, but, in this idealised data problem, it was misleading. When the graphs so produced did not match their earlier pencil and paper work the group used more meaningful strategies which involved connecting meanings drawn from the table of values to features of the graphical representation. They then continued using curve fitting and, although they did not derive an algebraic representation for the exponential function, they hit upon the correct exponential regression which made sense in the context of the problem. Earlier, the Israeli pupils had used a less sophisticated calculator, the TI 81, without regression. They had found the algebraic representations (but not without difficulty for the third exponential function) in order to create and interpret the graphical representations

The authors conclude that the Canadian students used the calculator both mechanically and meaningfully, with the search for meaning carrying them to a successful conclusion. Their use was influenced by the calculator, together with their previous learning experiences of using regression for handling real life data. However, the affordances of the technology (the regression facility) may encourage them to approach all modelling in this inductive way. Unlike the Israeli group, they did not produce an algebraic model first. Using recursion all the time may prevent them from reaching higher order modelling strategies. This is similar to the case of spreadsheets, where recursion is such an easy thing to do that students may remain with this as their only strategy and do not move on to finding generalisable algebraic expressions. The ICT may scaffold, but eventually restrict, their mathematics.

This study was assessed as having medium weight of evidence.

Isiksal and Askar (2005) compared the effect of Autograph-based instruction (ABI), spreadsheet (Excel) based instruction (SBI), and traditionally-based instruction on seventh-grade students' mathematics achievement and self-efficacy. Autograph is a piece of British mathematics educational software which offers a dynamic environment for Cartesian graphs as well as statistics and probability. Excel is an industry standard applications programme which can be used in mathematics learning. The study took place in a school in an upper middle class area in Ankara, Turkey. Three classes with 64 12-13 year-old students were randomly assigned to the three interventions.

The ABI group received two hours' instruction on how to use the software since it was new to them; the SBI group was not trained as they had used the software before in computer literacy classes. The ABI and SBI groups received instruction from the researcher and then worked without help (and largely individually) in the computer laboratory on the activity sheets. The control group was taught by their mathematics teacher.

A 20 short-answer type item Mathematics Achievement Test (MAT) 20 was administered as a pre- and post-test. The statistical analysis compared the significance of differences between gains in mean scores between the different groups.

The Autograph group gain scores were significantly higher than those of the Excel group. The traditional group scored significantly higher than the Excel group. There was no significant difference in mean scores between the Autograph and traditional groups, although the mean gain for the ABI group was higher than the TBI group. No significant difference was found between the gain scores of girls and boys.

The authors conclude that the higher scores for the Autograph group could be due to the design of the programme, which could have had a positive effect on the attitudes of the students, in turn leading to better performance.

This study was assessed as having high weight of evidence.

Mitchelmore and Cavanagh (2000) investigated student difficulties in using graphics calculators. Twenty five higher achieving students, aged between 16 and 17, from five metropolitan high schools in Sydney, Australia, were clinically interviewed as they undertook tasks designed to investigate misconceptions. Some students owned their own Casio Fx-7400G calculators, while others had limited access, and all students were inexperienced users of the technology.

The videotaped interviews captured students' responses, words and behaviour. Selected segments of the videos were transcribed and analysed task by task, then by themes, across the eight tasks.

Students exhibited many weaknesses. They had a limited concept of scale in graphs and poor understanding of the zoom function. They had difficulty making appropriate numerical approximations for the values they were looking for and did not necessarily link symbolic with graphical representation. Students based their answers on the visual image formed by highlighted pixels and did not realise the limitations of the visual image due to resolution. They did not link the visual representation of points of intersection on the screen with the coordinates of the point displayed at the bottom of the screen. They did not know how the GC produced the graph or the values of the coordinates. They liked to have the origin centrally positioned.

On the positive side, they could use the pixel groupings to deduce that the gradient of a parabola varied at different points along the graph. Students who owned their own GCs tended to have a better critical understanding of the calculator's output. (The Review Group noted that this might not be causal.)

The authors conclude that, in some cases, the calculator was exposing conceptual errors. The teacher could choose to smooth the path by ensuring the students do not confront these problems (e.g. by fixing scales). However, the teacher could use cognitive conflict as a teaching strategy, starting off by avoiding difficulties, but moving on to structure challenges which will expose misconceptions. To do this, teachers need to know how to use the calculators themselves.

This study was assessed as having medium weight of evidence.

Ninness et al. (2005) were unusual in working within a behaviourist theory of learning. They studied how the use of a computerised stimulus response programme can train learners to:

1. identify equivalent forms of formulae (from standard to factored forms and vice versa) for functions (square root and logarithmic)

2. identify the equivalence of formulae to graphs (from standard or factored formulae to graphs and vice versa) for functions (square root and logarithmic) which have been transformed by reflection or horizontal or vertical translations

3. do the same thing with novel functions (quadratic, cubic, tangent, sine, exponential), not included in the training programme

Ten participants aged 15 to 37 were tested to ensure they were not familiar with the transformation of function, using a pre-test. They then undertook a computer interactive training programme and took an assessment after each unit. Data was collected of the results of the assessments, errors made, and the number of times the assessments were taken before mastery.

Most participants mastered the computer interactive training sequence in a relatively small number of exposures (at worst on the third attempt). Following training, eight out of ten obtained at, or above, 85% accuracy on tests of novel relations, and six out of ten obtained 92.5% or better. One 15 year-old dyslexic boy made no errors on the assessment of trained relations and very few on the assessment of novel functions.

The authors conclude that this sort of training approach is a functional alternative to waiting for students to construct improved schema of mathematical understanding.

This study was assessed as having medium weight of evidence.

Sivasubramaniam (2000) compared English students' understanding of context-free graphs, Cartesian graphs in computer and paper media. She investigated students' ability to make interpretation at three different levels:

Level 1: Point-wise information on one graph or more than one graph, but only requiring one length on each axis (e.g. reading co-ordinates)

Level 2: Comparison of changes in length on one axis in relation to changes in length on the other

Level 3: Comparison of rates of change

Her null hypothesis was that there would be no difference in performance between the group working with the computers and the group working on paper. Both groups worked in pairs through learning modules during a one-hour session, without explicit help from the teacher. Each group constructed graphs from tables of values and then worked through interpretation questions with the help of notes provided. The researcher used a randomised pre- and post-test, control group experimental design with matched pairs, with 202 pupils aged 14-15 from top sets in 101 matched pairs.

The same test was used for pre-, post- and delayed post-test. Twenty pupils, with a range of attainment from10 matched pairs, were interviewed after the post-test. Test data gave information of assessment performance and which questions at which levels were attempted. Interview data revealed understanding at different levels of interpretation and the ability to justify and explain solutions. The test consisted of 28 cloze form and multiple choice items. The interview required students to think aloud while interpreting graphs. Their responses were probed and they then explained their answers for the post-test. The interview investigated the methods and the reasoning behind their methods.

The results indicated that the computer medium aided the development of interpretative skills more than the paper medium. Both groups of pupils improved from the pre-test to the post-test and on to the delayed post-test. The percentage increase for the computer group from pre- to post-test was 34.2%, and from post-test to delayed post-test 40.6% Corresponding figures for the pencil group were 24.9% and 25.2%. The increase for the computer group from the post to delayed post test is statistically significant ($p=0.043$) but not for the paper group. The effect size for matched pairs is 0.44. This size in the medium range suggests that the null hypothesis is unlikely to have been falsely rejected. Interviews showed that the paper group demonstrated more confusion in their explanations of interpretations and that the computer pupils were more likely to get questions on rates of change of gradients (i.e. level 3 thinking) correct.

The author argues that the computer directs the pupils' attention to interpretation rather than the construction of the graphs because the instant construction of the graph allowed virtually all the time available for interpretation. This efficient and accurate construction of the graphs on the computer has the potential for students' existing schema to be altered to accommodate new information, whereas the paper medium only provided opportunities for reinforcement. For the paper group, switching from construction to interpretation may have blurred the pupils' graphical perception. She concludes that the computer medium provided appropriate scaffolding for the development of schema for graph interpretation.

This study was assessed as having high weight of evidence.

Yerushalmy (2000) investigated the following:

1. students' methods when solving function-based problems involving introductory algebra when graphing technology was available

2. the development of the students' concept of function and how this concept impacted on their problem solving strategies

He used a non-standard longitudinal case study (over three years) of a pair of lower achieving

Israeli pupils using a high achieving pair for some comparison, gathering data through clinical interviews.

In the first interview, the pair tried to solve the linear breakeven situated problem using a table of values, but it is not clear that they were able to solve the problem. The higher attainers tried to use algebraic symbolism, but abandoned it and solved the problem numerically.

In the second interview, the pair solved the problem using a table filled in recursively and drew a graph by hand, again using a recursive strategy moving up at a constant rate. Initially they avoided the software as they would have needed algebraic symbolism to use it. Later, one boy did insert two correct symbolic expressions, but he did not go on use them. They noted a point in the table at which values seemed to 'switch' and zoomed in on this using smaller increments of the independent variable. The higher attaining pair did derive a symbolic expression and began to use the software, but lost confidence in their symbolism and fell back on an arithmetical method for solution.

In the third interview, the pair formed two correct symbolic expressions, equated them and tried to solve the resulting equation. They made a mistake with algebraic manipulation and then made a sketch graph to describe the situation. This led them to use the Function Graphing software. They typed in the equation, which the software then represented with a graph of the two intersecting lines. The higher attaining pair solved the problem quickly using algebraic techniques.

The problem-solving took the students a long time as they moved between representations and strategies. Their concept of function developed slowly over the three years but they found it difficult to attend to the abstract expressions and keep the problem in view. When they worked numerically and graphically, they were better able to keep the mathematics and the problem both in sight. For the case study pair, the use of the computer was delayed until the second half of the problem-solving process for the second and third interviews. The high attainers used the computer earlier because they were comfortable with using algebraic expressions and their later solutions were algebraic.

The authors conclude that the complexity of helping students to value algebraic symbols may take more than just bridging between representations. The choice of function as the main algebraic concept does not remove the difficulty of algebra, but it is conducive to an inquiry-based approach using accessible problems which can result in sustained work.

The observations lead to the conclusion that students will not become dependent on the software.

This study was assessed as having high weight of evidence.

4.4 Synthesis of evidence

In order to address the review question, it was necessary to decide:

- whether there was evidence of gains in understanding
- which aspects of multi-representations and interpretations of graphs were enhanced
- the classroom conditions under which these gains were achieved - this included any specific considerations which teachers needed to make in order for pupils to use the technology appropriately.

The synthesis will focus on the 12 studies from the in-depth review which are judged to have medium or high weighting overall, excluding the one study which was judged to be of low weighting in terms of design and focus for the review question.

There is a body of evidence from the studies included in the in-depth review that the use of graphing software, whether on graphics calculators or computers, did contribute to the development of pupils' understanding of functions. Improvements in understanding were gauged in some studies by improved performance on assessments, giving statistical evidence of gains as well as some indication of where the gains were made. Some studies of the work of whole classes and case studies of individuals or pairs of pupils elicited qualitative evidence, giving insight into the nature of these understandings. In probing the nature of the understandings, there is some evidence of limitations in understanding and of some of the difficulties which the use of ICTs posed for the learners. Some account also needs to be taken of the different ways of working which may have impacted on the students' development of understanding.

In the following sections, details of the tasks and assessment items are given as these provide a very precise idea of the mathematics involved and the demand for the learner.

4.4.1 Evidence of gains in understanding of functions

Some of the studies used a pre-test, post-test design, and some used a delayed post-test to measure gains in understanding. These need to be broken down into the aspects of functionality which were tested, as described in the different studies.

1. Godwin and Sutherland (2004) gave a diagnostic assessment to one of the classes before and after the design initiative, which involved using Omnigraph on computers. Pupils were asked to plot or sketch the graphs of three different

quadratic functions:

$$y = x^2,\ y = x^2 + 3 \text{ and } y = (x - 4)^2$$

In the pre-design initiative, 13.7% of the class of 51 were able to plot these correctly, and 55.6% after the design initiative either plotted or sketched the graphs correctly.

2. Gray and Thomas (2001) used a pre-, post- and delayed post-test, with an intervention involving students using graphics calculators, to assess the following:

- understanding of conservation of a linear equation by the addition of either an mx or c term
- the ability to solve an equation presented in one representation using another
- the relation between tabular, symbolic and graphical representations

Overall the students did better on the post-test (N=17, pre-test mean = 4.09, post-test mean = 6.12, t = 2.74, p<0,01 and there was some evidence of sustained improvement (N=17, pre-test mean = 4.09, delayed post-test mean = 5.38, t = 1.81, p< 0.05).

3. Hegedus and Kaput (2003), using graphical software on connected computers, found statistically significant increases in both the mean scores of all the students (N= 24, pre-test mean 42.7%, post-test mean = 65.9%, Cohen d effect 1.60, Hake's gain 0.42) and the two subgroups (N=10, pre-test mean 52.2%, post-test mean 76.8%, Cohen d effect 1.78, Hake's gain 0.5), (N=14, pre-test mean 37.7%, post-test mean 62.0%, Cohen d effect 1.91, Hake's gain 0.36) with significant improvements on items involving

- interpreting line graphs
- interpreting $y = mx + c$ as m and c vary
- interpreting slope in real life situations
- interpreting slope as rate
- generating and interpreting families of functions
- interpreting non-linear relationships (not a focus of the intervention)

4. Sivasubramaniam (2000) made a direct comparison between pupils' understanding of graphs in computer, and paper and pencil-based media. The results indicated that the computer medium aided the development of interpretative skills more than the paper medium. Both groups of pupils improved from the pre-test to the post-test and on to the delayed post-test.

The percentage increase for the computer group from pre- to post-test was 34.2% and from post-test to delayed post-test 40.6%.

Corresponding figures for the pencil group were 24.9% and 25.2%.

The increase for the computer group from the post- to delayed post-test is statistically significant (p=0.043), but not for the paper group. The effect size for matched pairs is 0.44.

5. Isiksal and Askar (2005) compared the performance of three groups, two taking computer-based instruction and one traditional-based instruction without technological tools. The autograph group (ABI) and traditional groups (TBI) had significantly greater mean scores than the spreadsheet group (SBI) and the autograph group had significantly greater mean scores than the traditional group.

Group	**N**	**Mean gain score**	**Standard deviation (SD) of gain scores**	**Mean difference**	**P**
ABI	21	11.86	11.17	ABI-SBI= 9.14	0.012*
SBI	21	2.71	10.17	TBI-SBI=8.60	0.017*
TBI	22	11.32	8.42	ABI-TBI= 0.54	0.983

* The mean difference is significant at the 0.05 level.

4.4.2 The nature of the understandings

It is clear that representing a function graphically is needed to see the effect of transformations, and the relationship between families of functions. Graphical approaches can also be used as an alternative to solving equations algebraically. The advantage of digital technology over time-consuming paper and pencil constructions is that graphs can be produced very quickly, giving instant feedback on the effect of changes.

Doerr and Zangor (2000), researching two whole classes with the same teacher, found students making use of visualisations in this way with their graphics calculators:

1. Students were able to fit graphs to data plots using previous knowledge. The students who recognised the class of functions to which the set belonged and then varied parameters were successful. However, without an initial recognition of the nature of the function, some students were misled by the regression facility, which may give a good fit for a limited range of values. This needed to be a meaningful process rather than random fitting.

2. Students understood that a quick judgment about the shape of the graph was not enough to

decide the nature of the function. In particular, they were alert to the possible confusion of an exponential with a quadratic function.

3. Students understood that, with tables of values, when there were repeated values, ‘there was something in between’. They then moved from using the table of values to construct an algebraic expression and, from this, the continuous graph. This gave them the behaviour of the functions between discrete values.

4. Students understood how to use graphs to solve equations. They chose a visual method because it was more meaningful for them.

5. Students were helped to understand the non-uniqueness of algebraic representations and stimulated to check the algebraic equivalence of functions by comparing their graphs.

The Doerr and Zangor finding about solving equations is corroborated by the Friedlander and Stein (2001) study of six pairs of students who had access to paper and pencil methods, spreadsheets, graphing software on the computer and a algebraic symbol manipulator (Derive). They were successful in using a graphical approach to yield a solution to quadratic and simultaneous equations. They preferred this visual approach to the algebraic symbol manipulator, because they felt it did not help them to understand the solution. Although they were able to use a graphical approach to solve a linear equation, nevertheless they preferred a paper and pencil approach.

Gray and Thomas (2001) had more mixed findings than Doerr and Zangor (2000) and Friedlander and Stein (2001), when they probed the students’ ability to solve equations involving inter-representational linking. Five students out of 17 showed some representational fluency, but others were unable to make connections between different representations.

For example, they were given a graph of a function and asked to use it to solve an equation where they needed to introduce a value or new function: for example, using the graph of y = x2 – 2x to solve x2 – 2x = 3 or x2 – 2x = x. Many were not able to use the given graph and resorted to drawing up tables of values from the symbolic representation and then trying to use these to solve the equation. Although they were successful at identifying one or two solutions, their approach revealed a point-wise discrete view of function rather than a holistic object view.

In contrast to all the tasks in the studies so far, the longitudinal case study by Yerushalmy (2000) gives data on a pair of lower attaining students working on contextualised problems. The problems, which were similar over the three years, were of the type:

A factory owner can give a bonus in the form of 450 IS per person or an additional amount equal to 1/5 of his regular salary. Find a way to represent the cost of both methods in order to help the owner choose the better bonus strategy.

The lower attaining students initially worked arithmetically using tables of values to compare values in two functions. They were reluctant to use the software because they found difficulty deriving a symbolic expression which they could then use to draw a graph with the software. Over the period of three years, they moved to a point at which they could combine their arithmetical approach to the problem with the symbolism required to use the graphing software to find the solution to two intersecting linear equations.

In Borba and Confrey’s (1996) study, comparing visual feedback with the symbolic form of the function equation stimulated the case study student to find a convincing explanation for his incorrect prediction. The software, Function Probe, allows a function to be created in different ways: that is, using algebraic symbolism, plotting point by point, sketching using the mouse like a pen. The graph of the function can be transformed directly using the mouse and the corresponding symbolic formulae either hidden or revealed. When Ron transformed the graph of y = x2 + 3x + 5 by a horizontal translation of 5 units to the right he expected the new equation to be y = (x + 5) 2 + 3 (x + 5) + 5 rather than the y = (x - 5) 2 + 3 (x - 5) + 5 which was revealed. He resolved the contradiction to his own satisfaction with the aid of a mediating metaphor of rubber sheets, one with a representation of the axes and the other with a representation of the curve.

Visualisation also featured strongly in the study by Godwin and Sutherland’s (2004). In one of the two classrooms, we have evidence of pupils being able to predict the graphs of functions like y = x + 3, y = x -1, y = x – 2 having discussed the prototype graph of y = x, and recognising that the graphs were all parallel with a gradient of 1. They were also able to give examples e.g. y = -x + 4, y = 2/x that were not parallel to the first family of graphs. Later, pupils were able to say that the gradients of graphs in the family y = 2x, y = 3x, y = 4x and y = 8x became progressively steeper. They could also use the trace function and the calculator display to find coordinate points which could be used to plot graphs by hand. This showed an understanding of the graphs as objects but also a point wise understanding of the function.

In whole class activity in the second classroom, there is evidence of students being able to predict the graphs of y = x2 + a and y = (x + c) 2 and y = -(x + c) 2 for positive and negative values of a and c, and then predicting the graph of y = (x + 2) 2 + 3 before going on to investigate general graphs of this form.

In the linked Godwin and Beswetherick (2002) study, one case study student following up the whole class work above produced anchor graphs (e.g. y = x, y = x2), prescribed graphs given by the teacher (e.g. y = 3 + x2), and experimental graphs in response to open questions such as the following:

> *What changes and what stays the same when you change the 'a' in y = - (x + a) 2?*

After the lessons, Kay understood the behaviour of quadratic graphs under varying conditions; that is, the effect of the various parameters on the behaviour of the graphical representation. She was also able to reproduce a graph using the graphic calculator having been given a sketch of the graph.

When comparing paper and pencil instruction with computer instruction, the computer pupils in Sivasubramaniam (2000)'s study were more likely to get questions on rates of change of gradients (i.e. level 3 thinking) correct. The interviews showed that the paper group demonstrated more confusion in their explanations of interpretations than the computer group.

4.4.3 Difficulties and strengths with using graphics calculators

Although only two studies reported on the difficulties and strengths of using graphics calculators, this would seem to be an important section for practitioners and has therefore been picked out for special treatment.

The study by Mitchelmore and Cavanagh (2000) gave detailed evidence of the difficulties which students had in using a graphics calculator.

Common difficulties were as follows:

- Given the symbolic equation of a parabola, then presented with a portion of the curve on screen which looked straight, students were misled. They thought this was a straight line and did not refer to the equation.
- They found difficulty adjusting scales so that perpendicular lines appeared perpendicular on the screen.
- They found difficulty identifying intersection points when low resolution melded two lines together over a number of pixels, and they also had difficulty identifying turning points when the vertex was not given by a single pixel. When making such interpolations, they did not use the coordinate information displayed on the screen to help them.
- They found difficulty in changing the window settings in order to make a parabola appear horizontal on a screen and they were wedded to having the origin at the centre of the screen

Strengths were as follows:

- Students could zoom out to find intersection points which were not displayed on the original viewing screen.
- They were also able to explain how the arrangements of pixels on the parabola graph indicate a changing gradient, unlike that of the straight line where the pixel arrangements were regular.

Hershkowitz and Kieran (2001) raise another difficulty which could arise when using graphics calculators. In this study, pupils had previously used the regression facility on the TI 83+ calculator when modelling real life data. When this approach was used with idealised data generated from investigating the growth patterns of areas of rectangles, linear regression was misleading. At first the students accepted a graph which seemed to fit their data, but, when they realised it did not fit all the values in their table, they tried other curves and, using trial and improvement, they were successful.

4.4.4 Ways of working

This section presents evidence of the ways in which teachers and pupils were working which may have impacted on the development of understanding. Clearly it is difficult to claim that these ways of working led directly to gains in understanding, any more than the technology itself will have led to gains. Rather, these ways of working are presented as contributing to the development of understanding.

Borba and Confrey (1996) argue that the case study student was aided towards his development of understanding by the careful construction of tasks and the ability of the researcher to listen closely. This was a one to one situation over a period of time which could not be matched in a natural classroom situation. However, Doerr and Zangor's (2000) study was of an experienced teacher with two classes where the teacher was confident and knew how to use the calculator herself. She encouraged the pupils to work meaningfully, to be alert to the limitations of the tool and methods, and to share their thinking in whole class discussion. Pupils were encouraged to take the lead in addressing the whole class. This was deemed to be necessary in supporting whole class understandings because, when pupils were in groups working with the graphics calculators, they tended to work individually and communication broke down.

A similar way of working was observed in the study reported by Godwin and Beswetherick (2002) and Godwin and Sutherland (2004). In the first of these papers, one of the teachers in the study started with whole class engagement, then students worked on the computers, and the teacher drew them back together for whole class discussion in the plenary. The tasks, which moved from relatively closed to more open, enabled the case study student, Kay, to develop an experimental approach in which she systematically investigated the behaviour of a general quadratic function.

The second study in this linked pair gives us data of lessons taught by two different teachers, and will be described here in detail, as there are important points to be drawn out from the ways in which they supported the learning of their pupils in their interactions and structuring of tasks. Rachel used graphics calculators because access to computer rooms was relatively difficult, whereas Rob had easy access to a computer lab. Both teachers used whole class and small group/individual work.

In whole class work Rachel worked from prototype functions (e.g. y = x) and gradually built up the range of examples. She built up families of lines which were parallel to this prototype, stressing the meaning and language of gradient as she progressed. She also encouraged the students to suggest non-examples: that is, straight lines which were not parallel to the original family of lines. When one student suggested a non-linear function, she guided him back to the focus of the lesson. In the whole class part of the lesson, Rachel used an overhead projector with acetates and the non-digital whiteboard to note down families of straight line graphs, associated language and the non linear example provided by the pupil. Pupils were then encouraged to experiment individually or in pairs with straight-line graphs. Those who were unsure of which graphs to experiment with were given suggestions. One boy immediately tried out the non-linear function he had suggested earlier, then continued to investigate the graphs suggested by the teacher.

Once this had been done, the students had to try to reproduce the graphs they had on their calculators in their books, using the trace function to find two pairs of coordinates which were then plotted and joined with a straight line. The plenary went over the work of the lesson, drawing on the students' own work, and drawing attention to the similarities and differences between their examples. The lesson was characterised by the following:

- a combination of structure and focus with experimentation
- the use of prototypes and related examples as well as non-examples
- the use of individual / pair work and whole class plenaries, with students' own work brought into the plenary
- connections made between the graphical representation and points on the graphs (thus addressing one of the difficulties raised in the Mitchelmore and Cavanagh study)
- use of digital and non-digital technology

In Rob's lesson, there was also a mixture of whole class and individual work. This differed from Rachel's in that students were asked to come out to the whiteboard and predict the shapes of graphs. Several attempts were made at this (correct and incorrect) before entering the equation into the computer and projecting the result. Unlike Rachel, the function equations involved decimal as well as integer parameters. The individual / pair work asked students to investigate the behaviour of graphs of the type y = (x + a) 2 + b. Some of the questions gave specific examples, whereas others were of a more general nature. Kay (described above in the linked study) and Jack worked differently on this, with Kay doing more exploration and Jack tending to stop with the specific examples given rather than investigating further. In the plenary, students had to write down the equations of displayed graphs and then sketch the graph of given equations. Rob then consolidated the learning by emphasising the effect of changing the values of a and b in terms of the direction and magnitude of horizontal and vertical shifts. Like Rachel, Rob also used

- a combination of structure and experimentation
- whole class and individual/pair work
- a prototype equation

and also

- used decimal values for constants

Building up collective knowledge within the whole class was a feature of the classroom connectivity in the study by Hegedus and Kaput (2003). In this situation, the pupils worked in pairs on the computer and passed their individual work to the teacher who could aggregate it and project it for the whole class onto a whiteboard display. They concluded that it was this way of working, combined with the graphing software, that accounted for the learning gains.

Further evidence of pupils in pairs or small groups working together in discussion was found in Gray and Thomas (2001), Hershkowitz and Kieran (2001), and Yerushalmy (2000), although the ways in which the teachers intervened and worked with the whole class are not made explicit.

In the Friedlander and Stein (2001) researcher manipulated intervention, students worked in pairs but were asked to solve a sequence of two single equations and two systems of equations, using as many different ways as they could, and selecting from four possible tools (paper and pencil, computerised graph-plotter, spreadsheet and algebraic symbol manipulator). Although they lacked some pre-knowledge (how to solve a quadratic equation using the algebraic formula, how to use a spreadsheet to solve algebraically presented equation), they used all the tools at some point to solve the equations and were able to evaluate the relative merits of the tools.

4.5 In-depth review weight of evidence results

Main paper	Component A Trustworthiness of the study in answering the study's questions	Component B Appropriateness of the design and analysis for the review question	Component C Relevance for the focus of the study for the review question	Composite D Overall weight taking into account of A, B and C
Borba and Confrey (1996)	Medium	Medium	Medium	Medium
Doerr and Zangor (2000)	High	High	High	High
Friedlander and Stein (2001)	Medium	Medium	High	Medium
Godwin and Beswetherick (2002)	High	High	High	High
Godwin and Sutherland (2004)	High	High	High	High
Gomes-Ferreira (1998)	Low	Medium	Low	Low
Gray and Thomas (2001)	High	High	Medium	High
Hegedus and Kaput (2003)	High	High	High	High
Hershkowitz and Kieran (2001)	Medium	High	Low	Medium
Isiksal and Askar (2005)	High	High	High	High
Mitchelmore and Cavanagh (2000)	High	Medium	High	Medium
Ninness et al. (2005)	High	Low	Low	Medium
Sivasubramaniam (2000)	Medium	High	High	High
Yerushalmy (2000)	Medium	High	High	High

Familiarity with, and accessibility to, ICT tools was another aspect of ways of working. Gray and Thomas (2001) attribute some of their lack of success to students having a relatively short time to work with their graphics calculators and not being able to take them home. Kay, the successful student in the Godwin and Beswetherick (2002) study, came from an ICT rich home background, with a father who worked in ICT and encouraged his daughter to use ICT at home. In the study identifying student difficulties with using a graphics calculator, Mitchelmore and Cavanagh (2000) found that students who had their own calculators had a better critical understanding of how to interpret the output compared with students who borrowed the school calculators.

4.6 Nature of actual involvement of users in the review and its impact

The membership of the Review Group includes a variety of user groups, although the data extraction was undertaken by academics and researchers. Other user group involvement was largely through email and informal contacts at conferences, and through publicising the work of the Review Group through subject and professional associations, organisations and societies. In addition, papers based on this systematic review have been, and will be, presented at a variety of seminars, workshops and conferences. Digests of the key findings and implications for policy and practice will be drawn to the attention of different user groups. The initial stage of dissemination has largely been directed at academics, teacher educators, researchers and policy-makers, but it is intended to widen the dissemination through the use of websites and articles in magazines and newspapers. It is too early to comment on the likely impact that this review will have on policy and practice.

4.7 Summary of results of synthesis

Gains in understanding

Three studies give evidence of general gains in interventions each using one type of ICT (Godwin and Sutherland, 2004; Gray and Thomas, 2001;

Hegedus and Kaput, 2003). One study indicates that pupils working in the computer medium performed better than those in the paper and pencil medium, although both made gains in graphical interpretation (Sivasubramaniam, 2000). One study evidences differences in gains according to the type of software, and importantly that an intervention not incorporating technology was more effective than one of the interventions incorporating ICT (Isiksal and Askar, 2005). This points to the design of the particular software and the way it is introduced to the pupils as being of importance.

Nature of understanding

There is evidence of some students successfully using visualisation with graphing software to fit graphs to datasets, to solve equations and to transform functions (Borba and Confrey, 1996; Doerr and Zangor, 2000; Friedlander and Stein, 2001; Godwin and Beswetherick, 2002; Godwin and Sutherland, 2004). In terms of interpreting graphs of rates of change, there is evidence that pupils working in a computer environment reached higher levels of thinking and were able to explain their thinking better than pupils working in a paper and pencil medium (Sivasubramaniam, 2000). There is also some evidence of lower attaining students preferring to work arithmetically with tables of values and only later moving to integrate the tables of values with computer generated graphs (Yerushalmy, 2000). There is also some evidence of pupils having difficulty with moving between symbolic, tabular and graphical forms when solving equations (Gray and Thomas, 2001). Some of these differences may be accounted for by differences in the tasks and whether the tasks were context-free or contextualised.

Difficulties of working with graphics calculators

There is evidence that students do not always know how to use the technology, interpret ambiguities in the output and exercise critical judgment when using some of the facilities of advanced calculators (Hershkowitz and Kieran, 2001; Mitchelmore and Cavanagh 2000). These studies are of relevance to our review question, because they show that the learner has to learn how to use the tool critically before it can be used effectively, and also that difficulties in using the tool effectively may be exposing conceptual difficulties.

Ways of working

There is evidence (Doerr and Zangor, 2000; Godwin and Beswetherick, 2002; Godwin and Sutherland, 2004) that students working together in small groups, and also working interactively with their teachers in whole classes, provided a learning environment in which the ICTs were harnessed effectively. The individual or small group use of the technology gave pupils a valuable opportunity for inquiry and experimentation (Gray and Thomas, 2001; Hershkowitz and Kieran, 2001; Yerushalmy, 2000). However, unless the teacher pulled this together and orchestrated whole class plenaries each individual student could develop their own idiosyncratic knowledge which may or may not accord with the common knowledge the teacher was intending to develop in the lesson. In one study, the connectivity of the computers allowed the teacher to demonstrate the work of individual pupils and build up collective knowledge in this way (Hegedus and Kaput, 2003). In the study in which one student worked with a researcher, the ability to listen carefully to the student was seen to be crucial (Borba and Confrey, 1996).

There is evidence from one study (Friedlander and Stein, 2001) that students can work with several different ICT tools and evaluate their respective advantages.

There is evidence from three studies (Gray and Thomas, 2001; Godwin and Beswetherick, 2002; Mitchelmore and Cavanagh, 2000) that students who use ICT out of school are better able to use it effectively within school.

CHAPTER FIVE

Implications

5.1 Strengths and limitations of this systematic review

One strength of this review is the publicly visible nature of the review procedure. The review has benefited from the collaboration of the Review Group, the EPPI-Centre and many other individuals who offered help and advice.

Another strength is the way in which it has focused on a specific area of the mathematics curriculum and so can give very precise details about the ways in which ICTs can develop understanding of functions.

The main limitations of the review are that the constraints involved in terms of time and cost inevitably mean that decisions about the focus of the review question and the review process have to be made to keep the review manageable. This meant the Review Group went on to do an in-depth study on two of the areas identified in the systematic map:

- the relationship between different ways of representing functions
- the interpretation of graphical representations of functions

Two other areas have not been subject to in-depth analysis:

- the development of algebraic symbolism
- operations on symbolic expressions

Another limitation of any review of this type is that the individual studies did not set out to answer the review question; they all have different designs and instruments. This is particularly relevant in terms of the tasks used to assess understanding where small differences may make a noticeable difference to the students' responses. Although all the studies in the in-depth review were considered to be evaluations, not all used control groups and not all compared different kinds of software and hardware. So there is evidence of gains but it is not always known if those gains could have been achieved without the use of ICT. Another limitation is the amount of evidence of the nature and quality of the teacher input. Most of the studies in this review concentrated on pupils and did not give detailed evidence of how the teachers supported their pupils in developing knowledge of the functional concept and knowledge of how to use the ICT tools. Any conclusions must therefore remain tentative.

The What Works Clearing (WWC) House reviews were not screened. Subsequently, the middle school curriculum review has been found to contain titles which may report potentially relevant interventions.

5.2 Implications

Interpretation and application of the results of this review requires further work by different users of research, but initial implications include the following:

5.2.1 Policy

The findings of this review offered some support for the use of ICTs in the teaching and learning of functions, an important part of learning algebra. This confirms the stance on the use of ICTs in the current National Curriculum Mathematics and Key Stage 3 National Strategy Framework for teaching mathematics: years 7, 8 and 9 (Department for Education and Employment, 2001), which will presumably be continued in future policy guidance. Some of the detail in the supplement of examples in the last document is very helpful in including the output of graphics calculators and spreadsheets to illustrate the following:

- how they can be used to generate sequences
- drawing out the meanings involved in interpreting the graphical output of functions
- making links between the graph and the coordinate pairs on the graphics calculator display as the trace function is used

There could be more on the following:

- the links between tables of values, symbolic representation and graphical representation - this could provide the bridge between functions and the solution of equations, which does not seem to be included in the section on graphs of functions
- critical use of graph-plotters, including how changing the scale can alter the appearance of the graph, how to use the zoom function, how to change windows, interpreting pixel displays

The National Strategy presently advocates the use of a three-part lesson, incorporating interactive whole class teaching. The research here shows that this structure could provide the right framework for a combination of individual/group work and whole class plenaries to allow the experimentation, direction and sharing which seems to maximise the potential of the ICTs. Time spent on constructing meanings in this way would seem to be particularly important in algebra, given the problems already outlined in the background.

Policy-makers have an important role in giving direction on the judicious use of different tools. Graphics calculators and computers can be used by pupils in individual or group activity; interactive whiteboards or computers with projectors can be used for whole class work. The evidence on ways of working in this review suggests that both have a place.

However, there is also evidence of a teacher using a non-digital whiteboard with an OHP to draw together effectively aspects of the pupils' work with graphics calculators. Digital and non-digital technologies can be used together to enhance learning. With increased use of the interactive whiteboard, it will be important to ensure that pupils still have the opportunities for autonomy and experimentation afforded by graphics calculators or class computers, and that personal constructions are shared with the whole class.

It is not possible to conclude from this review the degree of emphasis that teachers should place on the use of ICT in lessons.

Interpreting the curriculum in all its detail and developing the pedagogical practices clearly has implications for those involved in continued professional development policy.

5.2.2 Practice

This review supports the use of ICTs in developing understanding of functions, but the teacher has a pivotal role in structuring and supporting the learning, so any recommendations have to take account of the teacher's role in mediating the learning, and the teaching and learning context. Simply using ICT will not guarantee that students make more learning gains than using traditional paper and pencil methods.

Teachers need to be confident users of the technology themselves, although relatively straightforward starting points can stimulate rich activity. The teacher needs to be aware of how the scale, window and resolution may present misleading images. One way of overcoming these difficulties would be to smooth the path for students by setting the scale and window for them. Another way is to use cognitive conflict, to present students with a puzzling image (e.g. part of a parabola which looks like a straight line because of the choice of scale, two lines which do not cross within the set window) and encourage them to work through their misconceptions. Students need to be alert to these potential sources of confusion and given good access to the technology so that they develop familiarity.

An effective method for studying families of functions and exploring transformations is to start with a prototype function expressed symbolically, generate similar examples and also non-examples, relate the symbolic expression to the graph, and find a way of describing the family or transformation using mathematical language. Giving students some room to experiment with more open questions in this process can be productive.

Teachers need to help students make links between symbolic, tabular and graphical output by making these links explicit. A common approach to graphing functions is to start with a symbolic expression, make a table of values and plot these by hand. This can give students a point-wise view of a function, a process to be done rather than an object in its own right. Graphical software, on the other hand, takes the plotting away from the learner and presents the graph as an object which can be explored. This is very important when investigating families and transformations, and checking whether functions are equivalent; however, when solving equations, a point-wise view is also important, as the coordinates of specific points on the graphs will give solutions. It is important then that these links are made explicit and reinforced when working within any one of the representations. The review indicates that a full understanding of the links between different representations may take time and may be facilitated by regular access to the technology.

One message that comes out of the review for teachers is to encourage meaningful activity by moving between representations, discussing pupils' methods, and explaining thinking and interpretations.

5.2.3 Research

This review can contribute in two main ways to the research community: first in terms of methodology and secondly in terms of substance. Although most of the studies in the in-depth review were judged to be of high quality, there tended to be little justification given for the choice of sample and little attention to issues of reliability and validity at the data collection and analysis stages. It would also be helpful to declare what counts as success in an intervention. Some interventions, although taking place with a whole class, select single students or pairs for report. Whilst this gives a valuable in-depth picture of the potential of ICT; it is known how typical these responses were or why these pupils were selected for report. This is not to say that small sample studies are not valuable; small sample studies can give a valuable in-depth picture. However, more detail about the participants in the sample would enable the reader to gauge the limits on generalisability, and provide a useful starting point for large scale evaluation. One of the studies, while reporting statistical gains overall, was cautious in claiming too much for the intervention because only a minority of students attained multi-representational fluency. Other researchers, however, may have claimed this as a success.

In terms of substance, there is a need for more studies of different types probing students' understanding of functions within an ICT environment. In particular, teachers need to know more about the areas in which they need to provide carefully structured support in order to make full use of the ICT tools. Studies could include more comparative work, with larger samples, investigating the relative merits of different software and ICT tools. While there is some evidence of difficulties with graphics calculators, there is no comparative evidence of difficulties with graphing software used by individuals/small groups on computers, or graphing software used on inter-active whiteboards and/or computers with projectors used with the whole class.

More in-depth studies are needed of teachers and pupils working in the naturalistic setting of the classroom setting, and more in-depth probing of students' understanding, using similar tasks in clinical interviews.

Researchers could also follow up the potentially relevant interventions in ICT and algebra contained in the middle school curriculum review of the What Works Clearing (WWC) House:

(http://www.w-w-c.org)

CHAPTER SIX

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Appendix 1.1: Authorship of this report

This work is a report of a systematic review conducted by the Mathematics Education Review Group

The authors of this report are:

Maria Goulding (Department of Educational Studies, University of York)
Chris Kyriacou (Department of Educational Studies, University of York)

They conducted the review with the benefit of active participation from the members of the review group.

For further information about this review, please contact:

Maria Goulding and Chris Kyriacou
Department of Educational Studies
University of York
York YO10 5DD
ck1@york.ac.uk
mg28@york.ac.uk

For further information about the work of the EPPI-Centre, please contact:

EPPI-Centre
Social Science Research Unit
Institute of Education, University of London
18 Woburn Square

Tel: +44 (0)20 7612 6397
Fax: +44 (0)20 7612 6800
E-mail: EPPIAdmin@ioe.ac.uk

Review group

Mathematics Education Review Group

Patti Barber (Department of Primary Education, Institute of Education, University of London)
Peter Bland (Bootham School, York)

Lynne Carruthers (Fulford School, York)
Dr Robert Coe (Centre for Evaluation and Monitoring, University of Durham)
Dr Ann Dowker (Department of Experimental Psychology, University of Oxford)
Ann Gannon (Department of Educational Studies, University of York)
Maria Goulding (Department of Educational Studies, University of York)
Professor Charles Hulme (Department of Psychology, University of York)
Dr Qaimah Ismail (Department of Educational Studies, University of York)
Professor Barbara Jaworski (Department of Mathematics, Agder University College, Norway)
Dr Keith Jones (Department of Education, University of Southampton)
Dr Chris Kyriacou (Department of Educational Studies, University of York)
Ann MacNamara (National Numeracy Strategy, London)
Dave Miller (School of Criminology, Education, Sociology and Social Work, University of Keele)
Anne O'Connor (Wilberfoss Primary School, Wilberfoss, York)
Alison Robinson (Department of Educational Studies, University of York)
Dr Tim Rowland (Department of Education, University of Cambridge)
Dr Sue Sanders (Department of Education, University of Swansea)
John Sharpe (City of York LEA Advisory Service)
Professor Peter Tymms (Centre for Evaluation and Monitoring, University of Durham)

Advisory group membership

The membership of the Advisory Group is the same as the Review Group. There was an initial meeting in January 2006 to discuss the background and possible focus of the review, followed by email discussions to decide upon the review question. However, other individuals (teachers, researchers, policy-makers) with an interest in the review question were also invited to comment on the work of the Review Group at appropriate times. This was largely done through email and through discussions at conferences. In particular, the membership of British Society for Research into Learning Mathematics (BSRLM) were contacted at regular intervals and were considered to be an expert group which provided additional input.

Thanks are given for additional advice from the following:

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Jennny Gage (University of Cambridge)
Dr Dave Hewitt (University of Birmingham)
Dr S Kenniwell
Dr Jenny Houssart (The Open University)
Professor Celia Hoyles (Chief Adviser for Mathematics, DfES)
John Monaghan (University of Leeds)
Professor Dave Pratt (University of Warwick)
Professor Jim Ridgway (University of Durham)
Professor Ken Ruthven (University of Cambridge)
Professor Michael Shayer (King's College, University of London)
Alison Clark Wilson (West Sussex Institute of Education)
Professor Dylan Wiliam (Board of Science Education, Washington, USA)

The Review Group met again in July 2006 to discuss work in progress, in particular the keywording process. After keywording, the Review Group was consulted in the process of narrowing down the review question. A further meeting in November was cancelled because it was difficult to find a meeting date for a sufficiently large group to attend a viable meeting.

Conflict of interest

There were no conflicts of interest for any members of the Review Group.

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The Mathematics Education Review Group and this review are part of the initiative on evidence-informed policy and practice at the EPPI-Centre, Social Science Research Unit, Institute of Education, University of London, funded by the Department for Education and Skills (DfES). This particular review was commissioned and was funded by the Training and Development Agency for Schools (TDA).

Appendix 1.2: Inclusion and exclusion criteria

For a paper to be included in the systematic map, it had to satisfy the following inclusion criteria.

Inclusion criteria

- Must be an empirical study of the effects of ICTs, as defined for this review, in mathematics teaching
- Must be a study of the effects of using different ICTs, as defined for this review, on understanding in algebra, as defined for this review
- Must focus on students up to the age of 16
- Must be in mainstream school setting
- Must be an evaluation study
- Must be in English and published in a professional or academic journal, or presented at an academic conference between 1996 and 2006

These inclusion criteria were reformulated hierarchically as exclusion criteria on scope, study type, date and type of publication.

Exclusion criteria

Exclusion on scope

A study will be excluded if it is:

1. Not an empirical study of ICTs used in teaching/learning of mathematics (e.g. studies of ICT used only in assessment of mathematics are excluded)
2. Not focusing on the specified ICTs (i.e. ICTs included are small programs, programming languages, such as Logo, spreadsheets and graph plotting software, computer algebra systems, ILS (independent/individual learning systems), interactive whiteboards (IWBs) and other projection equipment, stand-alone computers, graphical calculators, data loggers but not internet, videoconferencing, broadcast and video film)
3. Not focusing on the effects on understanding algebra as defined for this review (e.g. probability, statistics and calculus are not included even though algebra may be used in these areas)
4. Not focusing on children or young people up to the age of 16

Exclusion on study type

A study will be excluded if it is:

5. A study categorised according to the EPPI-Centre current classification as

A description

B exploration of relationships

D methodology

E review

or a collection of articles. (Some databases have single entries for collections or conference papers.)

Exclusion on date and type of publication/source

6. Not published in an academic journal or presented at an academic conference between 1996 and 2006.

Appendix 2.2: Search strategy for electronic databases

ERIC was searched via Cambridge Scientific Abstracts on 6 July 2006.

The following strategy identified 264 records:

1. KW = ICT* or (INFORMATION COMMUNICATION* TECHNOLOG*) or (INFORMATION WITHIN2 COMMUNICATION* TECHNOLOG*) or LOGO* or SPREADSHEET* or GRAPH* or (INTEGRATED LEARNING SYSTEM*) or (INDIVIDUAL LEARNING SYSTEM*) or (INTERACTIVE WHITEBOARD*) or (DATA LOG*) or (COMPUTER ALGEBRA SYSTEM*) or CAS

2. DE = ALGEBRA

3. 1 and 2

4. 3 not CALCULUS

Date range: 1986-2006

Limited to: Journal articles only; English only

BEI was searched via Dialog Datastar on 6 July 2006.

The following strategy identified 188 records:

1. KW = ICT$ or INFORMATION NEXT COMMUNICATION$ ADJ TECHNOLOG$ or INFORMATION ADJ COMMUNICATION$ ADJ TECHNOLOG$ or LOGO$ or SPREADSHEET$ or GRAPH$ or INTEGRATED ADJ LEARNING ADJ SYSTEM$ or INDIVIDUAL ADJ LEARNING ADJ SYSTEM$ or INTERACTIVE ADJ WHITEBOARD$ or DATA ADJ LOG$ or COMPUTER ADJ ALGEBRA ADJ SYSTEM$ or CAS

2. DE = MATHEMATICS ADJ EDUCATION or KW = ALGEBRA

3. 1 and 2

4. 3 not CALCULUS

Date range: 1986-2006

Limited to: English only

AEI was searched via Dialog Datastar on 6th July 2006.

The following strategy identified 154 records:

1. KW = ICT$ or INFORMATION NEXT COMMUNICATION$ ADJ TECHNOLOG$ or INFORMATION ADJ COMMUNICATION$ ADJ TECHNOLOG$ or LOGO$ or SPREADSHEET$ or GRAPH$ or INTEGRATED ADJ LEARNING ADJ SYSTEM$ or INDIVIDUAL ADJ LEARNING ADJ SYSTEM$ or INTERACTIVE ADJ WHITEBOARD$ or DATA ADJ LOG$ or COMPUTER ADJ ALGEBRA ADJ SYSTEM$ or CAS

2. KW = ALGEBRA

3. 1 and 2

4. 3 not CALCULUS

Date range: 1986-2006

Limited to: English only

Appendix 2.3: Journals and conference papers handsearched; source for citations

Journals

Educational Studies in Mathematics

International Journal for Technology in Mathematics Education (formerly International Journal for Computer Algebra in Mathematics Education)

Journal for Research in Mathematics Education

For the Learning of Mathematics

Mathematics Teaching

Mathematics in Schools

Micromath

Conference papers

Papers for the day conferences of the British Society for Research into Learning Mathematics (BSRLM)

Visions of Mathematics Education: Embedding technology in learning. Proceedings of the 7th International Conference on Technology in Mathematics Teaching. University of Bristol, 26-29 July 2005.

Source for citations

Ferrara F, Pratt D, Robutti O (2006) The role and uses of technologies for the teaching of algebra and calculus: ideas discussed at PME over the last 30 years. In: Gutierrez A, Boero P (eds) *Handbook of research on the psychology of mathematics education: past, present and future*. Rotterdam: Sense Publishers, pp 237-273.

APPENDIX 2.4 EPPI-Centre keyword sheet, including review-specific keywords

A1. Identification of report
Citation
Contact
Handsearch
Unknown
Electronic database (please specify)
. . . .

A2. Status
Published
In press
Unpublished

A3. Linked reports
Is this report linked to one or more other reports in such a way that they also report the same study?
Not linked
Linked (please provide bibliographical details and/or unique identifier)
. . . .
. . . .
. . . .
. . . .

A4. Language (please specify)
. . . .

A5. In which country/countries was the study carried out? (please specify)
. . . .
. . . .
. . . .

A6. What is/are the topic focus/foci of the study?
Assessment
Classroom management
Curriculum*
Equal opportunities
Methodology
Organisation and management
Policy
Teacher careers
Teaching and learning
Other (please specify) . . . .

A7. Curriculum
Art
Business studies
Citizenship
Cross-curricular
Design and technology
Environment
General
Geography
Hidden
History
ICT
Literacy – first language
Literacy further languages
Literature
Maths
Music
PSE
Physical education
Religious education
Science
Vocational
Other (please specify) . . . .

A8. Programme name (please specify)

. . . .

A9. What is/are the population focus/foci of the study?
Learners
Senior management
Teaching staff
Non-teaching staff
Other education practitioners
Government
Local education authority officers
Parents
Governors
Other (please specify) . . . .

A10. Age of learners (years)
0-4
5-10
11-16
17-20
21 and over

A11. Sex of learners
Female only
Male only
Mixed sex . . . .

A12. What is/are the educational setting(s) of the study?
Community centre
Correctional institution
Government department
Higher education institution
Home
Independent school
Local education authority
Nursery school
Post-compulsory education institution
Primary school
Pupil referral unit
Residential school
Secondary school
Special needs school
Workplace
Other educational setting (please specify)
. . . .

A13. Which type(s) of study does this report describe?
A. Description
B. Exploration of relationships
C. Evaluation
   a. naturally-occurring
   b. researcher-manipulated
D. Development of methodology
E. Review
   a. Systematic review
   b. Other review

Review-specific keywords

1. *On which aspect of understanding algebra does the study focus?*

The development of algebraic symbolism

The relationship between different ways of representing functions

The interpretation of graphical representations of functions

Operations on symbolic expressions

Not stated

2. *On which aspect of ICT does the study focus?*

Spreadsheets

Graphics calculators

Graph-plotting software on computers

Computer algebra systems

Symbolic manipulations using advanced calculators

Logo

Integrated / individualised learning systems

Electronic whiteboard

Other

3. *Does the study report what the ICT tool does and give details of activities undertaken by students?*

Yes

No

Appendix 4.1: Details of studies included in the in-depth review

Borba MC, Confrey J (1996) A student’s construction of transformations of functions in a multiple representational environment

Main aims of study	To investigate the feasibility of inverting the traditional approach to transformation of functions (from symbolism to tables of values to graphs) by going from graph to tables of values to symbolism using the Function Probe software.
Study research questions	Will the emphasis on visualisation allow students to move more easily into algebraic visualisation whilst maintaining visual meaning for the symbolism?
Study method	Evaluation: Naturally occurring
Key findings	Ron (the case study student) was able to make meaningful connections between the visual, tabular and symbolic representation. He was able to resolve the differences between his predictions and the computer feedback by introducing a mediating metaphor, that of a double rubber sheet and thread. This contrasted with the intentions of the software designers.
Main conclusions	This approach has potential, but much depends on the careful construction of tasks and the ability to listen to students. Visual reasoning is a potentially powerful form of cognition, and one which requires that students are provided with adequate time, opportunities and resources to make constructions, investigations, conjectures and modifications. The strength of the pupil’s investigation lies in his ability to co-ordinate visual actions with changes in other representations. The study supports the view that pupils can develop effective strategies of inquiry when presented with an environment supporting the use of multi-representations. New forms of representation change the mathematics to be taught; teachers thus have to revise their understanding of mathematics in the light of the pupils’ initiatives.
Weight of Evidence A (trustworthiness in relation to study questions)	**Medium trustworthiness**
Weight of Evidence B (appropriateness of research design and analysis)	**Medium**: The question assumes relatively standard learning conditions, i.e., in a classroom. This study was made under very well-resourced conditions and was of only one boy. The knowledge and skill of the interviewer may also not have been typical.
Weight of Evidence C (relevance of focus of study to review)	**Medium**: Similar comments to previous question.
Weight of Evidence D (overall weight of evidence)	**Medium**

Doerr H, Zangor R (2000) Creating meaning for and with the graphing calculator

Main aims of study	To explore how the meaning of the graphic calculator (GC) as a tool for learning was co-constructed by the teacher, and how the students used the tool to construct meanings.
Study research questions	How does the teacher's knowledge, beliefs and role affect her use of GCs in the classroom? How do students use GCs when learning mathematics? How do the teacher's role, knowledge and beliefs interact and relate to the student's GC use? What are the constraints of GC use?
Study method	Evaluation: Naturally occurring
Key findings	The role, knowledge and beliefs of the teacher and the nature of the tasks resulted in the emergence of a rich use of the GC. Students used the tool as a computational device, transformational tool, data collection and analysis tool, visualising tool and checking tool. The students were encouraged to work meaningfully, to be alert to the limitations of the tool and methods (e.g. relying on the appearance of the graph, relying on regression tools for curve fitting without mathematical justification) and to share their thinking in whole-class discussion. They were often invited to take the lead in addressing the whole class. As time went on, when in small groups, pupils tended to work individually, but when in whole-class discussion, the sharing supported whole-class understandings.
Main conclusions	The teacher created a rich learning environment in which the GC did not become the mathematical authority. The students were able to use the GC critically. This was due to the role, knowledge and beliefs of the teacher. The GC could lead to very individual constructions and pathways without the use of whole-class sessions where students shared their thinking.
Weight of Evidence A (trustworthiness in relation to study questions)	**High trustworthiness**
Weight of Evidence B (appropriateness of research design and analysis)	**High**: The research design allows for insight into how learners make use of the graphing calculator to make meaning of their tasks. The analysis highlights the ways they do this.
Weight of Evidence C (relevance of focus of study to review)	**High**: The focus of the study gives insight into the significant role and knowledge of teachers in promoting and encouraging students' successful and meaningful use of graphing technology.
Weight of Evidence D (overall weight of evidence)	**High**

Friedlander A, Stein H (2001) Students' choice of tools in solving equations in a technological learning environment.

Main aims of study	To explore: (i) students' solutions to equations when they have a choice of tools (paper and pencil and electronic tools); (ii) students' ability to choose, use and integrate various representations; and (iii) students' views on the tools.
Study research questions	How do students solve equations when they have a choice of tools (paper and pencil and electronic tools)? How much are students able to choose, use and integrate various representations? What are students' views on the tools?
Study method	Evaluation: Naturally occurring
Key findings	The students used an average of 3.8 methods per pair and used all available tools. They had low levels of success in solving the quadratic equation using paper and pencil but they had not been taught the algebraic formula before. There was a low frequency of spreadsheet use but they had not been taught explicitly on the course how to solve equations using spreadsheets. All pairs found the solution to each equation or pair of equations. Students tended to start using paper and pencil and then moved to using a computer tool. They preferred the paper and pencil method for the linear equation but the computer tools for the quadratic and simultaneous equations. They thought the symbolic manipulator was quick and easy but it did not help them understand the solution process. The advantage of the graph plotter was its transparency.
Main conclusions	The students were able to present the equations in various representations, move between tools and between representations and connect the outcomes. They had not been taught to do this before. They could give advantages and disadvantages for the tools, rather than straight preferences.
Weight of Evidence A (trustworthiness in relation to study questions)	**Medium trustworthiness**
Weight of Evidence B (appropriateness of research design and analysis)	**Medium**: The way the students interacted with the tools seemed to be spontaneous. We would need to know more detail of the rest of the programme to make some inferences about the sorts of learning experiences which enabled these students to work flexibly in this way. For instance there may have been a general stress on making meaning and justifying solutions in the programme as a whole.
Weight of Evidence C (relevance of focus of study to review)	**High**: The findings are helpful in showing average students' ability to accommodate the different ICT tools even when they are not sufficiently well versed about the ability of the tool, to move between tools and between representations and make connections between outcomes. This has implications for giving students new technological tools because this might suggest that the strength of students' subject knowledge might influence how well the tool benefits their learning.
Weight of Evidence D (overall weight of evidence)	**Medium**

Godwin S, Beswetherick R (2002) Reflections on the role of task structure in learning the properties of quadratic functions with ICT: a design initiative.

Main aims of study	To explore how one student involved in the research responded to the structures of the tasks set; her degree of experimentation; and what she learned.
Study research questions	What is the role of task structure in directing prescribed and experimental work?
Study method	Evaluation: Naturally occurring
Key findings	Kay (the case study student) was unable to plot any of the three graphs in the pre-initiative assessments, but she was able to sketch them afterwards. She demonstrated a good understanding of the behaviour of quadratic graphs under varying conditions. She was able to reproduce two graphs from sketches without any help, and two with some minor researcher hinting.
Main conclusions	Creating the right learning environment so that a relatively easy to use software like Omnigraph can result in successful learning requires planning and thought. This design used whole-class engagement at the beginning of the lesson where the teacher used the software with a data projector and an ordinary whiteboard; after that, the students worked on computers and then the teacher drew them all back together for a plenary. This mix, together with the range of closed and more open tasks on the worksheet allowed a mix of experimentation and prescription. In this way students kept to task but had a rich learning experience. The teacher felt he could open up some of his tasks more in future, having looked at the video data. He also report that Kay had enjoyed the work and that the tasks encouraged the students to reflect on their actions, to think and predict, enabling the students to gain conceptual insights.
Weight of Evidence A (trustworthiness in relation to study questions)	**High trustworthiness**
Weight of Evidence B (appropriateness of research design and analysis)	**High**
Weight of Evidence C (relevance of focus of study to review)	**High**
Weight of Evidence D (overall weight of evidence)	**High**

Godwin S, Sutherland R (2004) Whole-class technology for learning mathematics: the case of functions and graphs.

Main aims of study	To explore how teachers use digital tools for enhancing the learning of functions and graphs.
Study research questions	What were the similarities and differences in the learning experiences offered to pupils by the two teachers? What is evident from a comparison of the range of tools used by the two teachers; from a comparison of the way the teachers worked with their classes; and from a comparison of the pupils' ways of working with maths?
Study method	Evaluation: Naturally occurring
Key findings	**Understanding**: Both classes improved their understanding of functions during the course of the lessons. The two relatively low achieving boys focused upon in the analysis in Rachel's class, who experimented with their own ideas as well as Rachel's, made considerable gains. The calculator was a tool which enabled them to investigate in ways that would have been difficult with paper-based technology. Some students were less engaged with the mathematical activity. **Ways of working**: The two different teachers worked in different ways but both used prototype functions. Pupils within the same class experimented with the ICTs in very different ways. When pupils worked individually with the calculators, it was difficult to see what was on their screens, but there was collaborative mathematical talk. When one pair shared the calculator, the boy took possession of the calculator and their talk was social, not task related. **The knowledge community**: Rachel used the pupils' responses in whole-class interactions to build shared knowledge using the whiteboard and OHP. Rob used projected computer images in whole-class plenaries and encouraged pupils to come to the front of the class to make predictions which were then tested
Main conclusions	There is a tension between pupils experimenting with the technology and hence developing individual and idiosyncratic knowledge and the development of collective mathematical knowledge in the community of the classroom. Teachers have a considerable amount of choice in the way in which they use the technology with pupils and the way they interact with the class. Effective tools include digital and non-digital technologies.
Weight of Evidence A (trustworthiness in relation to study questions)	**High trustworthiness**
Weight of Evidence B (appropriateness of research design and analysis)	**High**
Weight of Evidence C (relevance of focus of study to review)	**High**
Weight of Evidence D (overall weight of evidence)	**High**

Gomes-Ferreira GV (1998) Conceptions as articulated in different microworlds exploring functions

Main aims of study	To explore the different ways students interact with three computer microworlds designed to explore functions.
Study research questions	How do students articulate knowledge of mathematical functions while interacting with different microworlds?
Study method	Evaluation: Naturally occurring
Key findings	In the Dynagraph (Parallel) microworld, students demonstrated a co-variational concept of function but line symmetry and periodicity were rarely identified. In the DG Cartesian microworld, they also developed a co-variational view. There were few attempts to link the representations in the DG Parallel microworld with existing school knowledge of functions. This was a new representation where students seemed free of previous conceptions. But when they did make the connections, as one pair did in the final interview, these were robust. When using DG Cartesian and Function Probe they did make connections with terms from their school mathematics. With Function Probe the students tended to explore and modify their existing conceptions.
Main conclusions	The conclusion largely reiterates the findings, but brings in a new point about the interactions between students, making them more precise in comparing two functions; however, no evidence of this has been presented in the paper.
Weight of Evidence A (trustworthiness in relation to study questions)	**Low trustworthiness**
Weight of Evidence B (appropriateness of research design and analysis)	**Medium**
Weight of Evidence C (relevance of focus of study to review)	**Low**
Weight of Evidence D (overall weight of evidence)	**Low**

Gray R, Thomas M (2001) Quadratic equation representations and graphic calculators: procedural and conceptual interactions

Main aims of study	To examine how the graphic calculator encourages cognitive links between different representations of the quadratic function: symbolic, tabular and graphical.
Study research questions	What is the students' ability to work within the symbolic representation (e.g., when linear and constant terms are added to both sides of an equation)? Could they solve an equation presented in one representation by using another? Could they relate processes for solving equations in tabular, symbolic and graphical representations?
Study method	Evaluation: Naturally occurring
Key findings	Students did better on the post test and there was some evidence of some sustained improvement. Students scored higher on the delayed post test than on the post test. There was no change in solution rate for the symbolically presented equations. In particular there was poor understanding of the balance mode of equations. Many students did not make connections between the different representations. They had a reliance on the symbolic form and a point wise process perspective of function. There was poor use of tables for solving equations. At least two students did relate the different representations.
Main conclusions	The use of the GC was not as successful at building up representational fluency as hoped and the students did not improve their understanding of some aspects of quadratic functions. Their interactions with each representation were largely process rather than object oriented. Their progress may have been impaired by not being able to take the GCs home. The authors remain convinced of the potential of GCs but feel they have not found the right pedagogical format.
Weight of Evidence A (trustworthiness in relation to study questions)	**High trustworthiness**: Even though there is insufficient description of the quality assessment issues in the report, the reported findings are well illustrated with the students' solution processes, which show the extent of their understanding of the relationship between the representations and of their understanding of the capabilities of graphic calculators.
Weight of Evidence B (appropriateness of research design and analysis)	**High**: The data that emerged from this research design has the potential to provide information on the value of graphic calculators in students' learning of quadratic equations, through its capabilities of multiple-representations.
Weight of Evidence C (relevance of focus of study to review)	**Medium**: (i) The students were not familiar with the graphic calculator before the research period. The familiarisation lesson and the rigour of the preparation lesson and teaching module may not have been sufficient for all students in the sample group to acquire the understanding to make connections between the different representations and to understand sufficiently and be confident with the capabilities of the graphic calculator. (ii) A similar group of students in year 11 might generate more significant results in terms of their ability to make connections and their usage of the graphic calculator in their tasks.
Weight of Evidence D (overall weight of evidence)	**High**

Hegedus SJ, Kaput J (2003) The effect of SimCalc connected classrooms on students' algebraic thinking

Main aims of study	To assess the effectiveness of a system where pupils work in pairs on computers on the algebra of change and variation but pass their individual work to the teacher, who can aggregate and display their work to the whole class.
Study research questions	Does classroom connectivity offer new and exciting pedagogical opportunities for teachers and significantly improve students' performance in core algebra topics over a short space of time?
Study method	Evaluation: Researcher-manipulated
Key findings	The connected SimCalc classroom had a significant effect on the students' learning. The effect size for both groups of students was extremely high (1.78, 1.91). The 9th grade students had a greater effect size (1.91). The 7th and 8th graders had a greater gain relative to their performance on the pre test. The gain was mainly based on the intervention, not related to previous knowledge. Item by item results: there were statistically significant gains involving interpreting graphs, interpreting y=mx+b as m and b vary, linearity, interpreting slope, and generating and interpreting families of functions. There was a statistically significant gain in an item which required students to interpret the graph of a quadratic function, although the intervention had concentrated on linear functions.
Main conclusions	By combining the dynamic SimCalc environment with classroom connectivity, students' performance on 10th grade algebra-related questions can be improved in a short period of time.
Weight of Evidence A (trustworthiness in relation to study questions)	**High trustworthiness**: But given the concentration on effect sizes and significance it would have been best to use a control group.
Weight of Evidence B (appropriateness of research design and analysis)	**High**
Weight of Evidence C (relevance of focus of study to review)	**High**
Weight of Evidence D (overall weight of evidence)	**High**

Hershkowitz R, Kieran C (2001) Algorithmic and meaningful ways of joining together representatives within the same mathematical activity: an experience with graphing calculators

Main aims of study	To find out how the students used the graphic calculators to solve a problem, and how they interweaved working in a mechanistic algorithmic way with working in a meaningful way.
Study research questions	How do students interweave two ways of using tool-based representatives: the mechanistic algorithmic way and the meaningful way?
Study method	Evaluation: Naturally occurring
Key findings	The Canadian group, using TI 83 plus calculators, first worked in a mechanical fashion using the misleading linear regression facility which they had been exposed to in previous work where curve fitting was used to help model real-life data. When the graphs so produced did not match their earlier pencil and paper work, the group used more meaningful strategies which involved connecting meanings drawn from the table of values to features of the graphical representation. They then continued using curve fitting and did not derive an algebraic representation for the exponential function, but they succeeded in reaching a solution which made sense in the context of the problem. The search for meaning was the guiding thread of the dialectical process between the mechanical and the meaningful. The Israeli pupils had used a less sophisticated calculator, the TI 81. They had found the algebraic representations in order to create and interpret the graphical representations.
Main conclusions	The Canadian students used the calculator both mechanically and meaningfully, with the search for meaning carrying them to a successful conclusion. Their use was influenced by the calculator, together with their previous learning experiences of using regression for handling non-idealised data. However, the affordances of the technology (the regression facility) may have encouraged them to approach all modelling in this inductive way. Unlike the Israeli group, they did not produce an algebraic model first; doing this all the time may have prevented them from reaching higher order modelling strategies. This is similar to the case of spreadsheets, where recursion is such an easy thing to do that students may remain with this as their only strategy and not move on to finding generalisable algebraic expressions. The ICT may scaffold but eventually restrict their mathematics.
Weight of Evidence A (trustworthiness in relation to study questions)	**Medium trustworthiness**: Some concerns about the sampling, the data collection methods and tools, and the data analysis process.
Weight of Evidence B (appropriateness of research design and analysis)	**High**
Weight of Evidence C (relevance of focus of study to review)	**Low**: More detail about the context, especially teacher interventions, would have helped us answer our review question better. This research shows how the students move between mechanical and meaning seeking activity, but not how the teacher can mediate.
Weight of Evidence D (overall weight of evidence)	**Medium**

Isiksal M, Askar P (2005) The effect of spreadsheet and dynamic geometry software on the achievement and self-efficacy of 7th-grade students

Main aims of study	To explore the effect of Autograph-based instruction (ABI), spreadsheet (Excel)-based instruction (SBI), and traditionally-based instruction (TBI) on 7th grade students' mathematics achievement and self efficacy.
Study research questions	What is the effect of Autograph-based instruction, spreadsheet-based instruction, and traditionally-based instruction on 7th grade students' mathematics achievement and self efficacy?
Study method	Evaluation: Researcher-manipulated
Key findings	The Autograph group gain scores were significantly higher than the Excel group. The Traditional group scored significantly higher than the Excel group. No significant difference in mean scores between the Autograph and Traditional groups, although the mean gain for the ABI was higher than the TBI group. No significant difference was found between the gain scores of girls and boys.
Main conclusions	The ABI group outperformed both the other groups, and were significantly better than the SBI group. The topics studied were related to visualisation - the design of the two programs could have accounted for the differences in gains. The gains could also have been due to greater self-efficacy in the ABI group, which may have been due to the more positive attitudes to Autograph.
Weight of Evidence A (trustworthiness in relation to study questions)	**High trustworthiness**
Weight of Evidence B (appropriateness of research design and analysis)	**High**
Weight of Evidence C (relevance of focus of study to review)	**High**
Weight of Evidence D (overall weight of evidence)	**High**

Mitchelmore M, Cavanagh M (2000) Students' difficulties in operating a graphics calculator

Main aims of study	To investigate student difficulties in using a graphic calculator in detail, and to distinguish the difficulties of using a graphic calculator arising from the lack of understanding of the technology from those which are caused by more fundamental, mathematical misconceptions.
Study research questions	What difficulties do students have in using a graphic calculator? Can we distinguish difficulties of using a graphic calculator arising from the lack of understanding of the technology from those which are caused by more fundamental, mathematical misconceptions?
Study method	Evaluation: Researcher-manipulated
Key findings	Students had a limited concept of scale in graphs and poor understanding of the zoom function. They had difficulty making appropriate numerical approximations for values they were looking for. They did not necessarily link symbolic with graphical representation. Students based their answers on the visual image formed by highlighted pixels, and did not realise the limitations of the visual image due to resolution. They did not link the visual representation of points of intersection on the screen with the coordinates of the point displayed at the bottom of the screen. They did not know how the GC produced the graph or the values of the coordinates. They liked to have the origin centrally posi0tioned. They could use the pixel groupings to deduce that the gradient of a parabola varied at different points along the graph. Students who owned their own GC tended to have a better critical understanding of the calculator's output.
Main conclusions	Some of the errors are conceptual and the calculator exposes them. The teacher could choose to smooth the path by ensuring that the students do not confront these problems, e.g. by fixing scales. The teacher could use cognitive conflict as a teaching strategy, starting off by avoiding difficulties but moving on to structure challenges which will expose misconceptions. Teachers need to know how to use the calculators themselves.
Weight of Evidence A (trustworthiness in relation to study questions)	**Medium trustworthiness**: Difficult to know whether the findings indicate misconceptions in interpreting graphic displays on the calculator or insufficient understanding of key ideas of graphs and the algebraic representation of functions.
Weight of Evidence B (appropriateness of research design and analysis)	**Medium**: It exposes the difficulties and suggests teaching approaches (e.g. cognitive conflict) to address them but does not evaluate these approaches.
Weight of Evidence C (relevance of focus of study to review)	**High**: Knowing about these errors is very important for practitioners.
Weight of Evidence D (overall weight of evidence)	**High**

Ninness C, Rumph R, McCuller G, Harrison C, Ford AM, Ninness SK (2005) A functional analytic approach to computer-interactive mathematics.

Main aims of study	To find out if using a computerised stimulus response computer program can train learners to: (i) identify equivalent forms of formulae (from standard to factored forms and vice versa) for functions; (ii) identify the equivalence of formulae to graphs (from standard or factored formulae to graphs and vice versa) for functions which have been transformed by reflection or horizontal or vertical translations; and (iii) to do the same thing with novel functions not included in the training program.
Study research questions	Can the participants be trained to match the graphs of transformed functions (reflections, vertical and horizontal shifts) to their corresponding formulae?
Study method	Evaluation: Researcher-manipulated
Key findings	Most participants mastered the computer interactive training sequence in a relatively small number of exposures (at worst on the third attempt). Following training, 8 out of 10 obtained at or above 85% accuracy on test of novel relations and 6 out of 10 obtained 92.5% or better. The 15-year-old dyslexic boy made no errors on the assessment of trained relations and very few on the assessment of novel functions.
Main conclusions	This sort of training approach is a functional alternative to waiting for students to construct improved schema of mathematical understanding.
Weight of Evidence A (trustworthiness in relation to study questions)	**High trustworthiness**: Analysis and exemplars address the aims of the study in a detailed and informed manner.
Weight of Evidence B (appropriateness of research design and analysis)	**Low**: We are looking for understanding; this approach is aiming for mastery.
Weight of Evidence C (relevance of focus of study to review)	**Low**: The evidence in the case of the one particular dyslexic 15-year-old is very interesting as it may indicate a very different teaching approach from a meaning-making approach for students with this special need. This is a very particular focus and a very small sample.
Weight of Evidence D (overall weight of evidence)	**Medium**

Sivasubramaniam P (2000) Distributed cognition, computers and the interpretation of graphs

Main aims of study	To examine students' understanding of Cartesian graphs in computer and paper and pencil media, focusing particularly on students' interpretation of graphs in the light of theories of distributed cognition.
Study research questions	Is there a difference in performance between the group working with the computers and the group working on paper?
Study method	Evaluation: Researcher-manipulated
Key findings	The computer medium aids development of interpretative skills more than the paper medium. Both groups of pupils improved from the pre test to the post test and on to the delayed post test. The percentage increase for the computer group from pre to post test was 34.2% and from post test to delayed post test 40.6%. Corresponding figures for the pencil group were 24.9% and 25.2%. The increase for the computer group from the post to delayed post test is statistically significant ($p=0.043$) but not for the paper group. The effect size for matched pairs is 0.44. This size in the medium range suggests that the null hypothesis (no difference) is unlikely to have been falsely rejected. Interviews showed that the paper group demonstrated more confusion in their explanations of interpretations and that the computer pupils were more likely to get questions on rates of change of gradients (i.e. level 3 thinking) correct.
Main conclusions	The computer directs the pupils' attention to interpretation rather than the construction of the graphs. They used all the time available for interpretation. The efficient and accurate construction of the graphs allows: existing schema to be reinforced; existing schema to be altered; existing faulty schema to be retained; and the construction of schema in the absence of existing schema. The paper medium only provided opportunities for reinforcement. Switching from construction to interpretation may have blurred the pupils' graphical perception. The computer medium provided appropriate scaffolding for the development of schema for graph interpretation.
Weight of Evidence A (trustworthiness in relation to study questions)	**Medium trustworthiness**
Weight of Evidence B (appropriateness of research design and analysis)	**High**
Weight of Evidence C (relevance of focus of study to review)	**High**
Weight of Evidence D (overall weight of evidence)	**High**

Yerushalmy M (2000) Problem solving strategies and mathematical resources: a longitudinal view on problem solving in a function based approach to algebra

Main aims of study	To analyse students' construction of function-based problem-solving methods in introductory algebra.
Study research questions	What characterises the development of the students' concept of function and how does this concept impact on their problem-solving strategies?
Study method	Evaluation: Naturally occurring
Key findings	The study used a case study of a pair of lower achieving pupils using a high achieving pair for some comparison. In the first interview the pair tried to solve the linear break-even situated problem using a table of values but it is not clear that they were able to solve the problem. The higher attainers tried to use algebraic symbolism but abandoned it and solved the problem numerically. In the second interview the pair solved the problem using a table filled in recursively and drew a graph by hand again using a recursive strategy moving up at a constant rate. The higher attaining pair did derive a symbolic expression and began to use the software but lost confidence in their symbolism and fell back on an arithmetic method for solution. In the third interview the pair formed two correct symbolic expressions, equated them and tried to solve the resulting equation. They made a mistake with algebraic manipulation and then made a sketch graph to describe the situation, leading them to use the Function Graphing software. The higher attaining pair solved the problem quickly using algebraic techniques.
Main conclusions	The problem solving took the students a long time as they moved between representations and strategies. Their concept of function developed slowly over the three years but they found it difficult to attend to the abstract expressions and keep the problem in view. When they worked numerically and graphically they were better able to keep the mathematics and the problem both in sight. The complexity of helping students to value algebraic symbols may take more than just bridging between representations. Observations made in the study lead to the conclusion that students will not become dependent on the software. The choice of function as the main algebraic concept does not remove the difficulty of algebra but is conducive to an inquiry-based approach and can result in sustained work.
Weight of Evidence A (trustworthiness in relation to study questions)	**Medium trustworthiness**: There is insufficient information on many aspects of the study such as the methods of data collection and analysis, and also the research question.
Weight of Evidence B (appropriateness of research design and analysis)	**High**
Weight of Evidence C (relevance of focus of study to review)	**High**
Weight of Evidence D (overall weight of evidence)	**High**

The results of this systematic review are available in four formats:

SUMMARY	Explains the purpose of the review and the main messages from the research evidence
REPORT	Describes the background and the findings of the review(s) but without full technical details of the methods used
TECHNICAL REPORT	**Includes the background, main findings, and full technical details of the review**
DATABASES	Access to codings describing each research study included in the review

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Evidence for Policy and Practice Information and Co-ordinating Centre (EPPI-Centre)
Social Science Research Unit
Institute of Education, University of London
18 Woburn Square
London WC1H 0NR

Tel: +44 (0)20 7612 6367

http://eppi.ioe.ac.uk/
http://www.ioe.ac.uk/ssru/

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